Computer Graphics

Lecture 7: Monte Carlo Integration

Kartic Subr

Using randomness

Georges-Louis Leclerc, Comte de Buffon
(1707-1788)

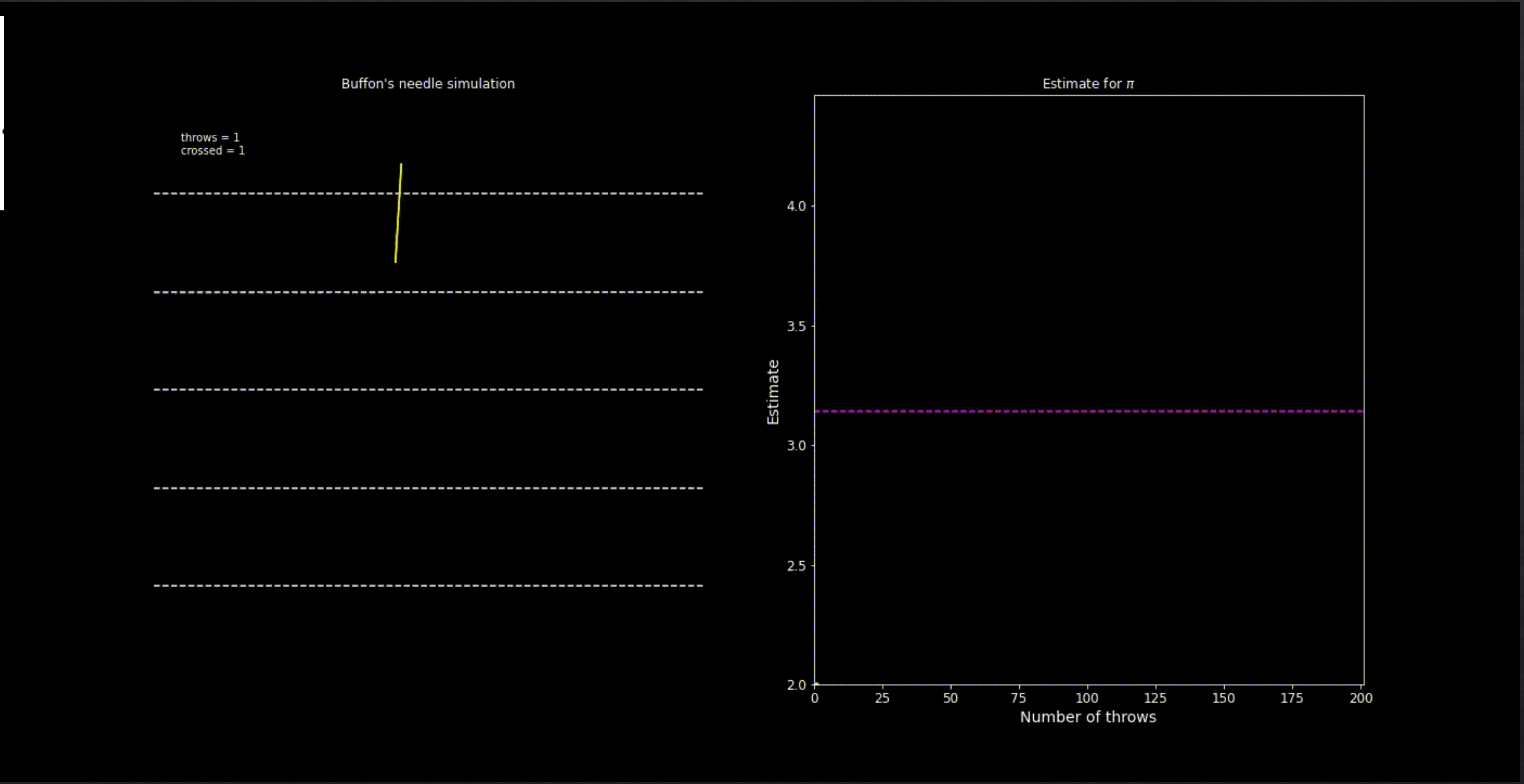
Using randomness

Georges-Louis Leclerc, Comte de Buffon (1707-1788)

Suppose we have a floor made of parallel strips of wood, each the same width, and we drop a needle onto the floor. What is the probability that the needle will lie across a line between two strips?

Using randomness

$$p = \frac{2}{\pi}\frac{l}{t}$$



https://people.orie.cornell.edu/sbanerjee/courses/orie4580f20/
http://dmcpress.org/cm/buffon/experiment.html

Random variables: refresher

$\underline{X}$

Expectation $\langle X \rangle = \int_a^b x \, p(x) \, dx$

Variance $\mathrm{Var}(X)$

$= (X - \langle x \rangle)^2$

$X \in [a, b]$

$X \sim p(x)$

Numerical integration using random samples

$$\langle x \rangle = \int \underline{x}\, p(x)\, dx$$

$$\langle \cancel{g} f(x) \rangle \doteq \int f(x)\, \underline{p(x)}\, dx$$

$$\int f(x)\, dx$$

$$\langle \underline{f(x)} \rangle = \int f(x)\, dx$$

average

From integrating circles to general functions

$$I = \int_0^1 f(x)\,dx$$

$$\boxed{x_i \sim U[0,1]}$$

$$\mu = \sum \frac{f(x_i)}{N}$$

$$\langle \mu \rangle = I$$

$$x_i \sim \underline{p(x)}$$

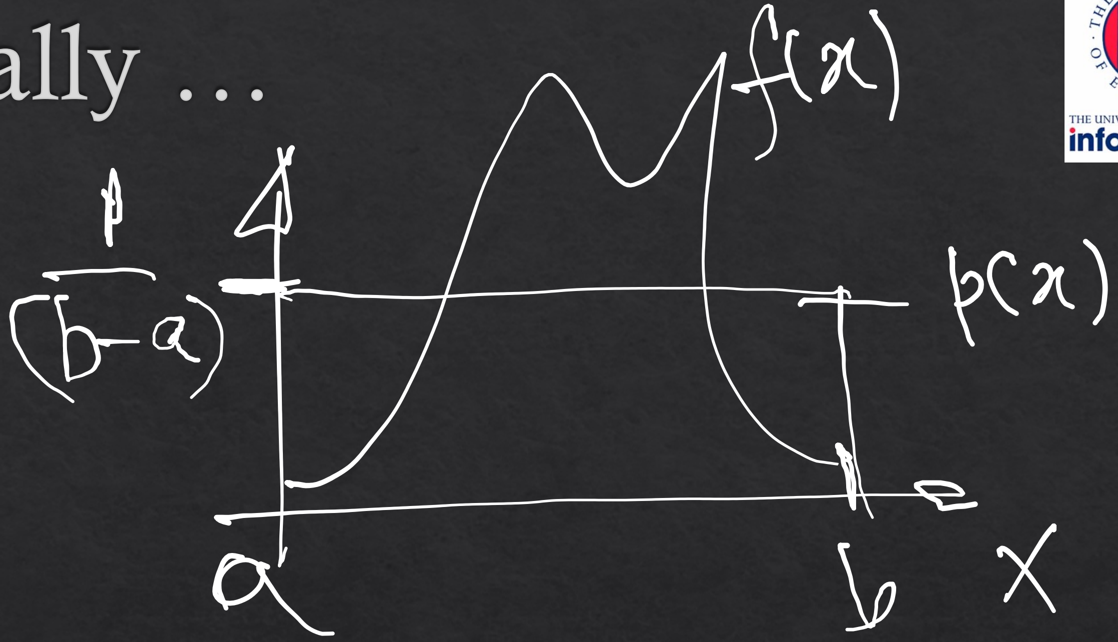
$$\langle f(x) \rangle = \underline{\int f(x) \cdot p(x)\,dx}$$

$$\langle g(x) \rangle = \int \left(\frac{f(x)}{p(x)} \right) \cdot p(x)\,dx$$

$$g(x) = \frac{f(x)}{p(x)}$$

Formally …

$$I \approx (b-a)\frac{\sum f(x_i)}{N}$$

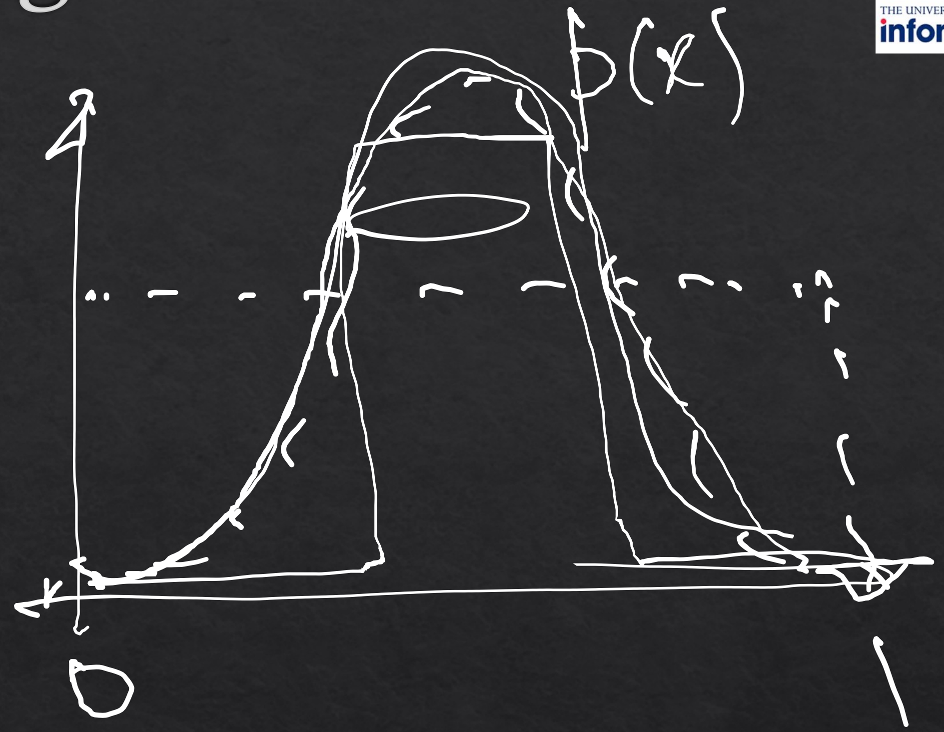


Importance sampling: intuition

$x_i \sim p(x)$

$I = \int_0^1 f(x)\,dx$

$\mu_p = \frac{1}{N}\sum \frac{f(x_i)}{p(x_i)}$



$\int \frac{f(x)}{\cancel{p(x)}}\,\cancel{p}(x)\,dx$