Computer Graphics

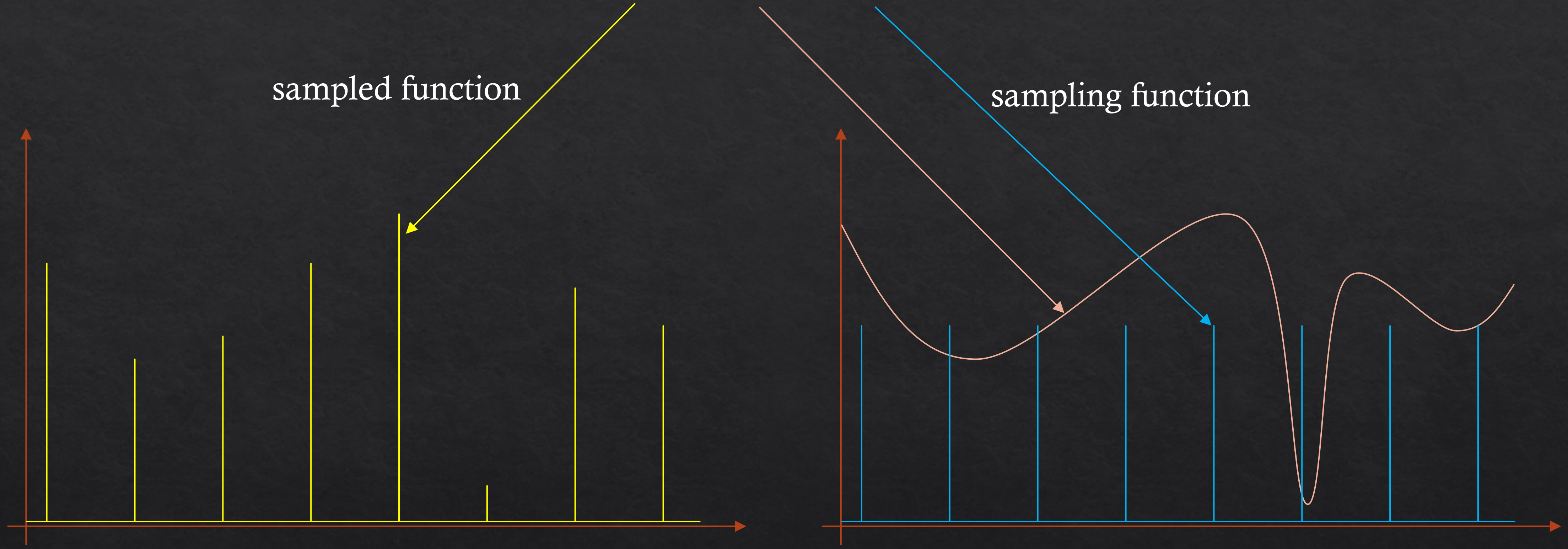
Lecture 12: Sampling II

Kartic Subr

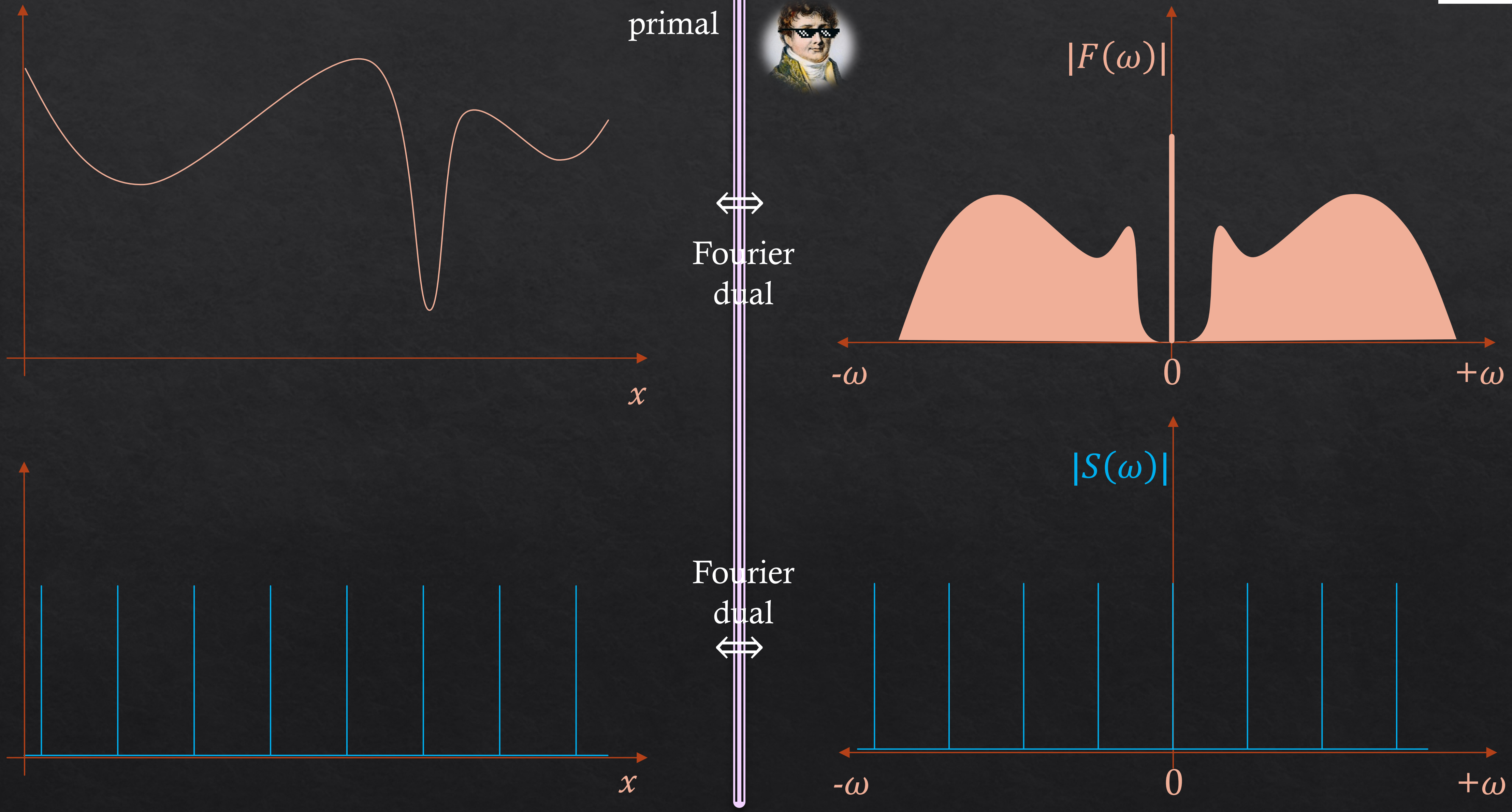
THE UNIVERSITY of EDINBURGH
informatics

Sampling = multiplication

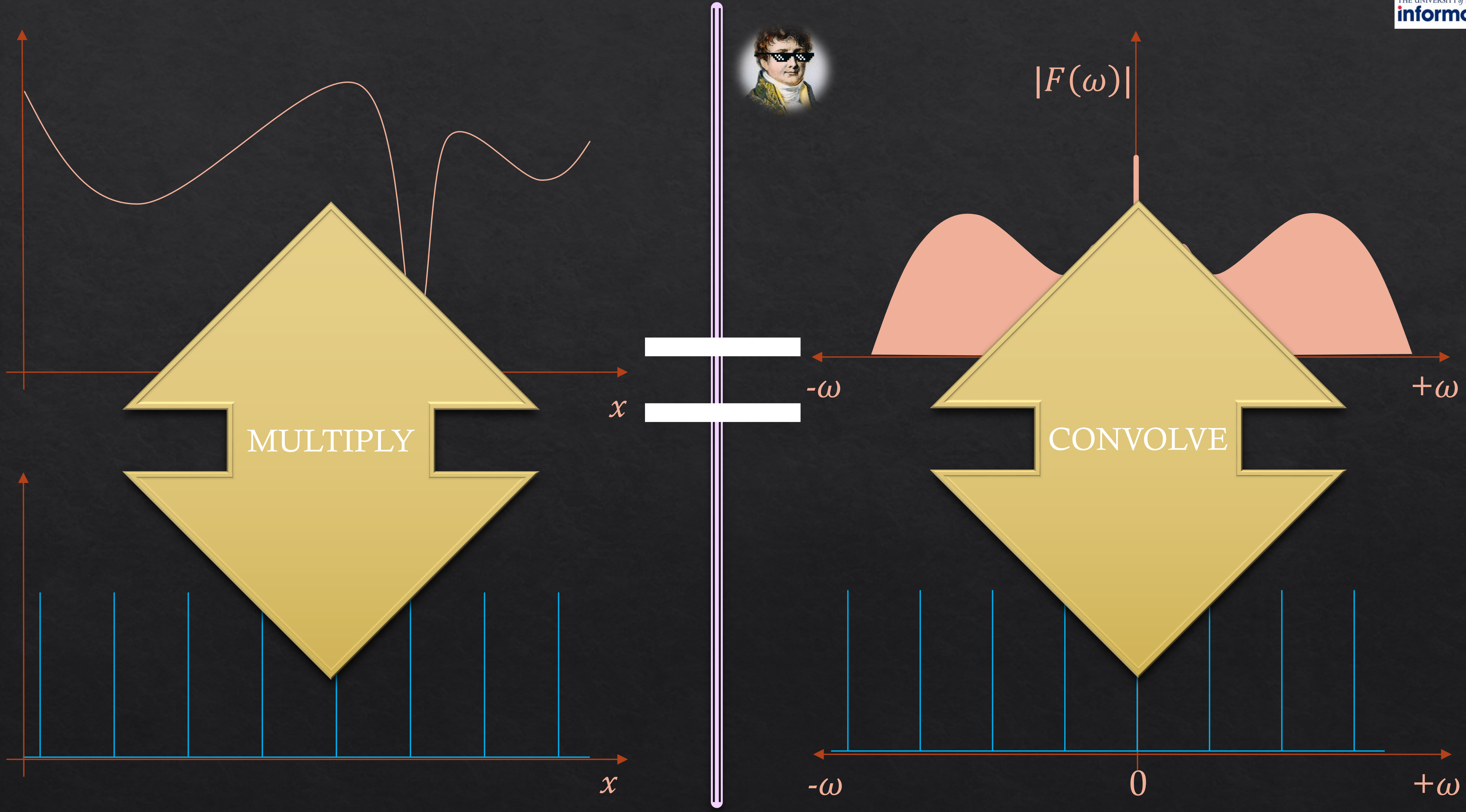
$$f_s(x) = f(x)\,.\,s(x)$$



Functions in Fourier domain

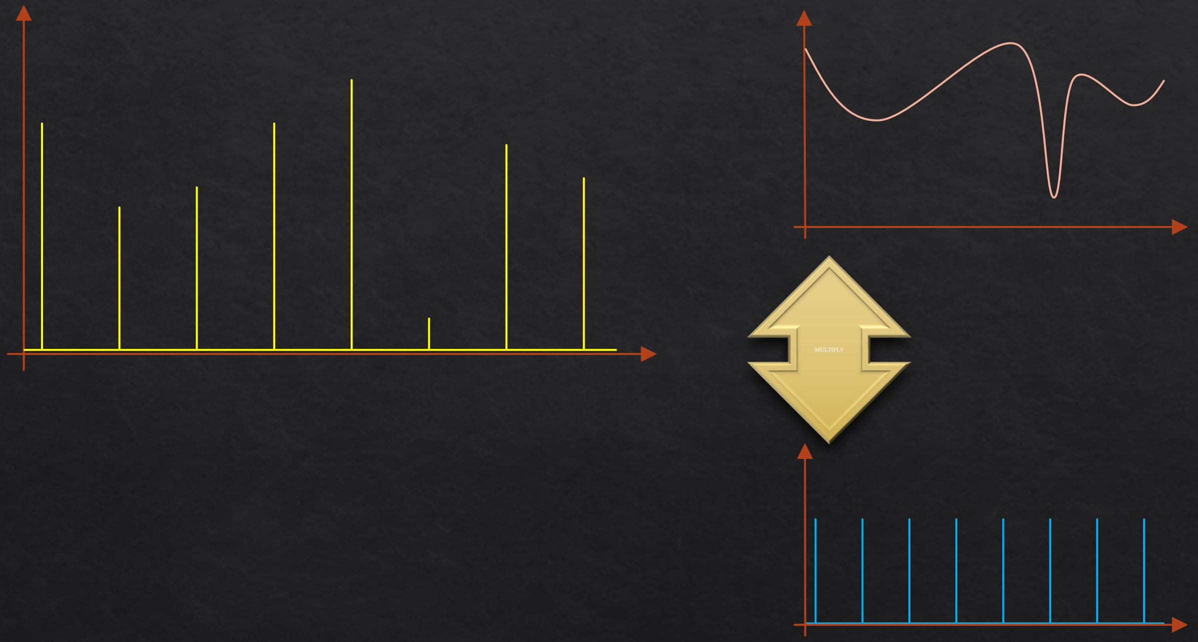


Convolution theorem

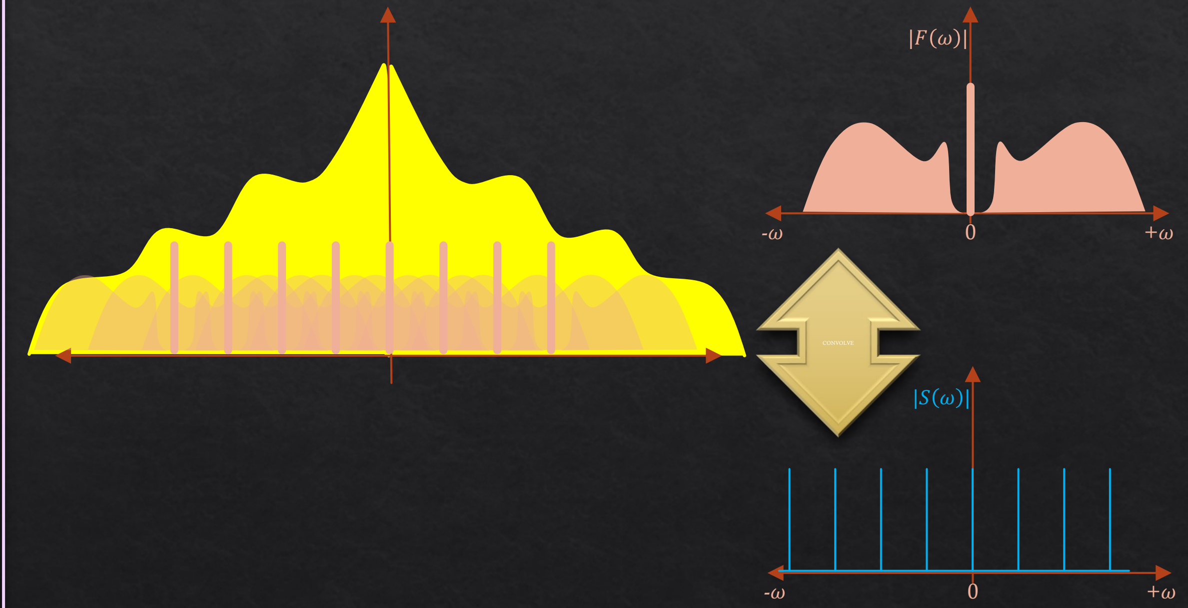


Sampling = convolution (Fourier domain)

$f_s(x) = f(x) \, . \, s(x)$

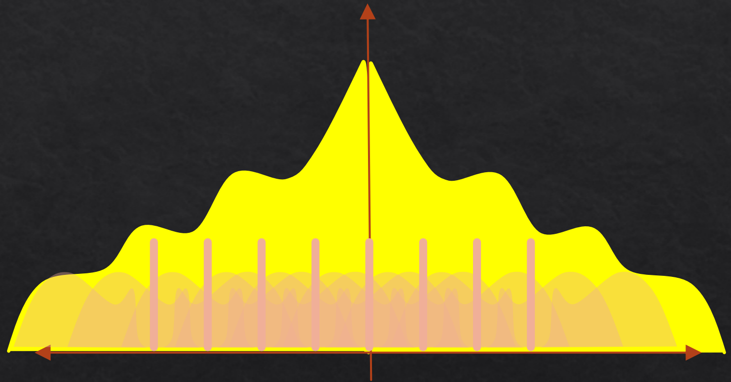


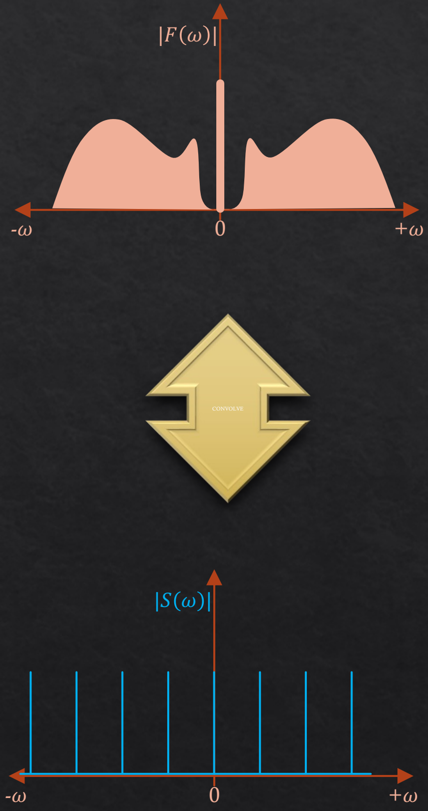
$F_s(\omega) = F(\omega) \otimes S(\omega)$



Convolution results in overlapping spectra

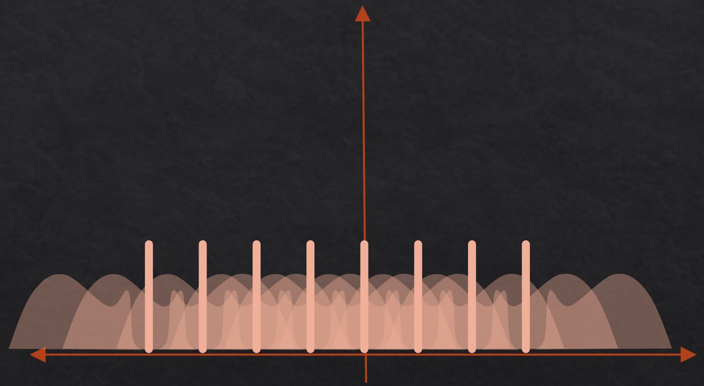
$$F_s(\omega) = F(\omega) \otimes S(\omega)$$

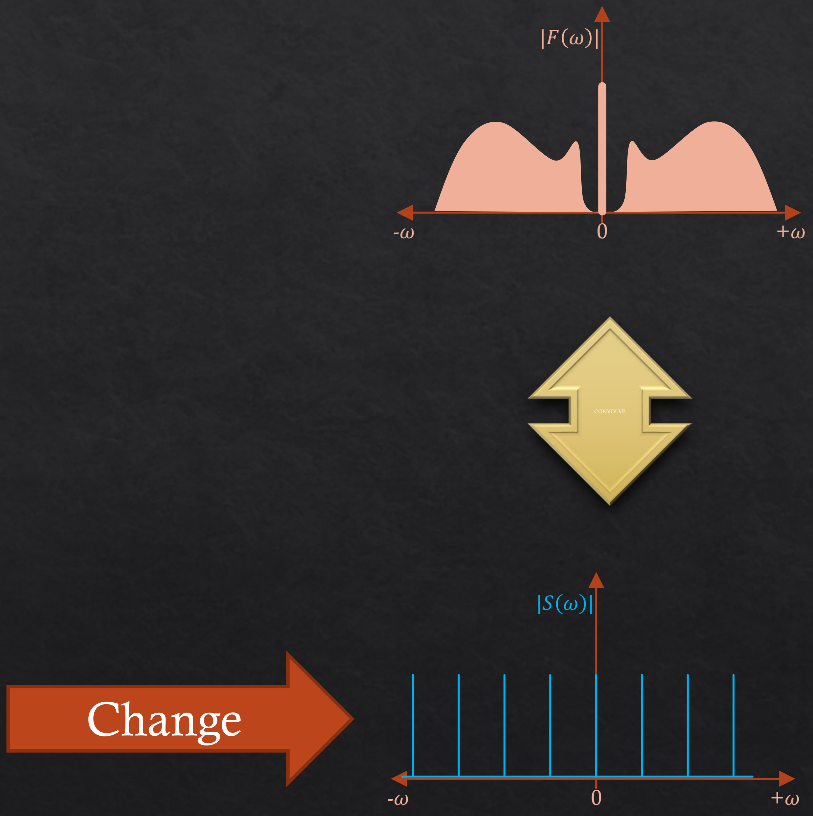




To move these away, change sampling function

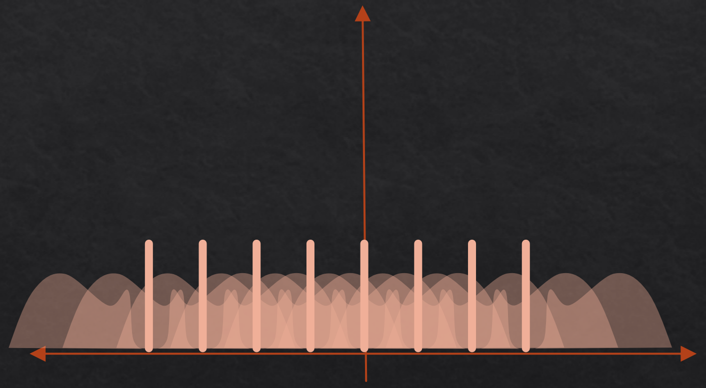
$$F_s(\omega) = F(\omega) \otimes S(\omega)$$

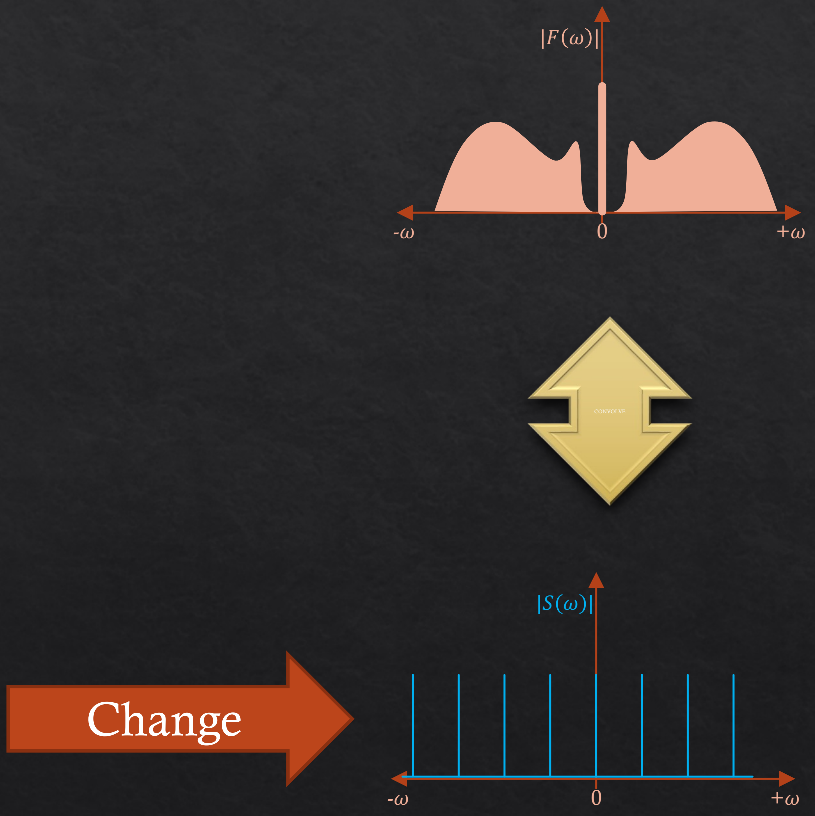




Stretching S to higher frequencies …

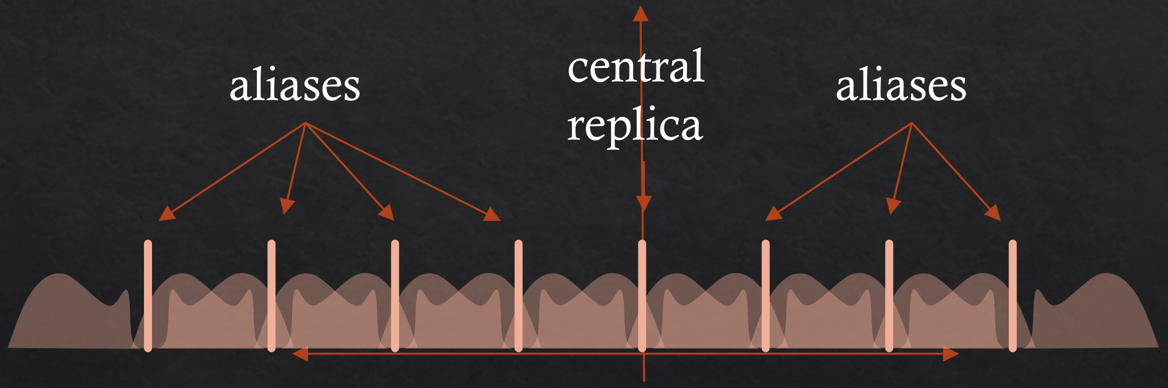
$$F_s(\omega) = F(\omega) \otimes S(\omega)$$

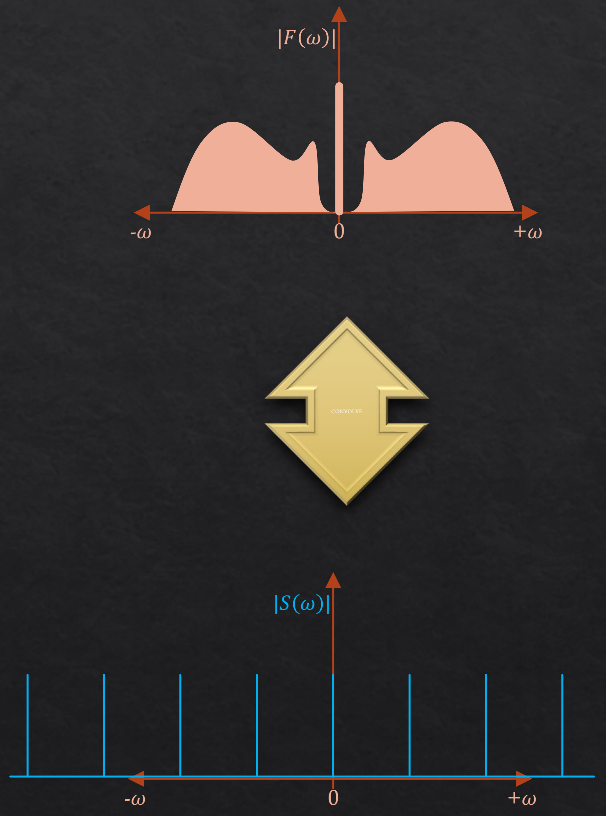




We can see the central replica and aliases

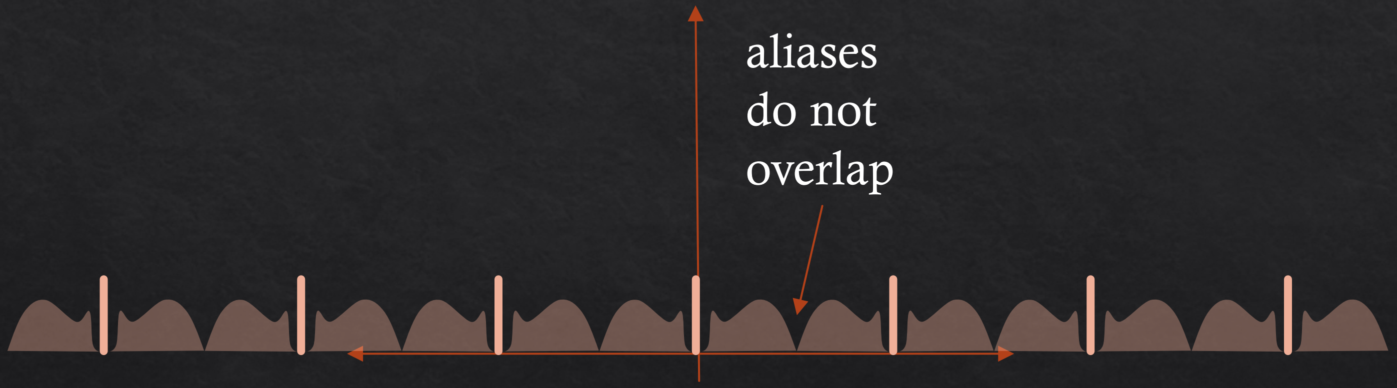
$$F_s(\omega) = F(\omega) \otimes S(\omega)$$

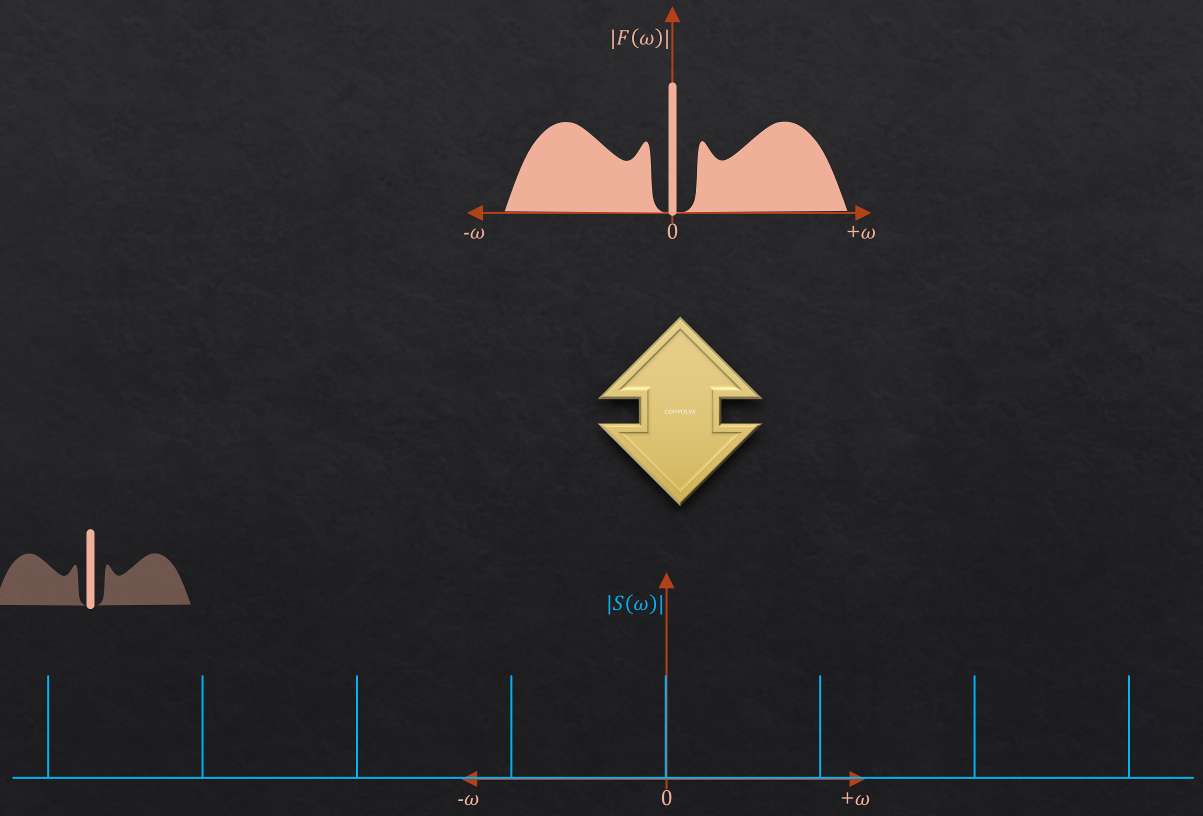




Pushing further removes overlaps

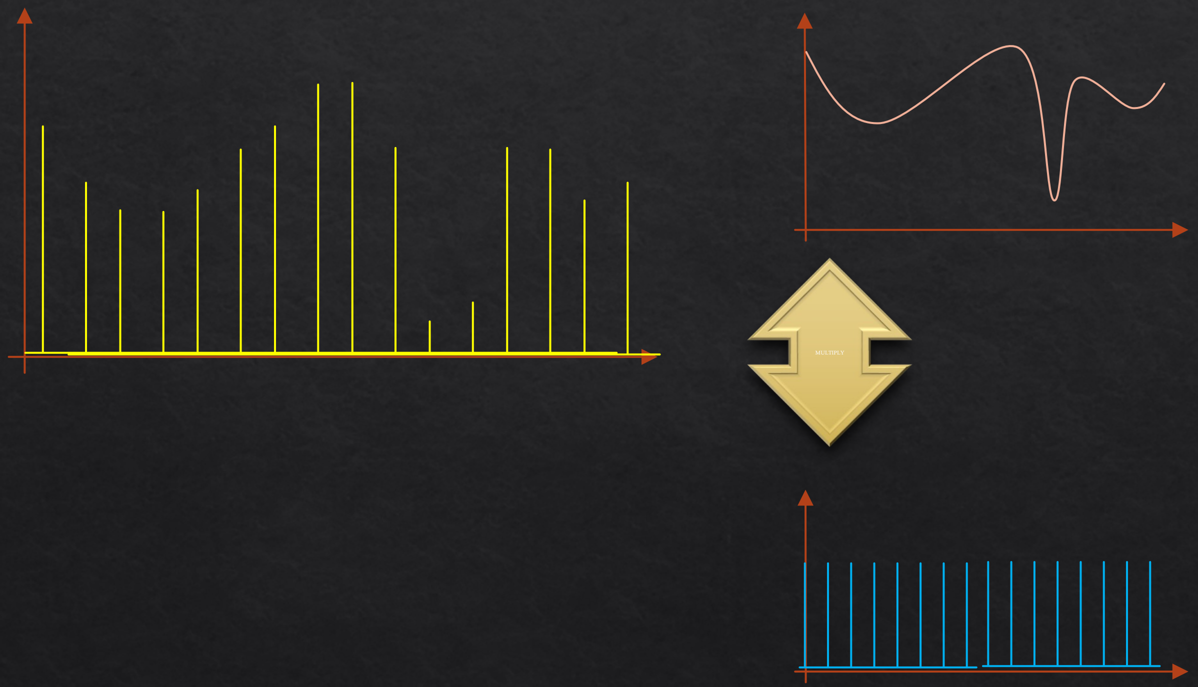
$$F_s(\omega) = F(\omega) \otimes S(\omega)$$

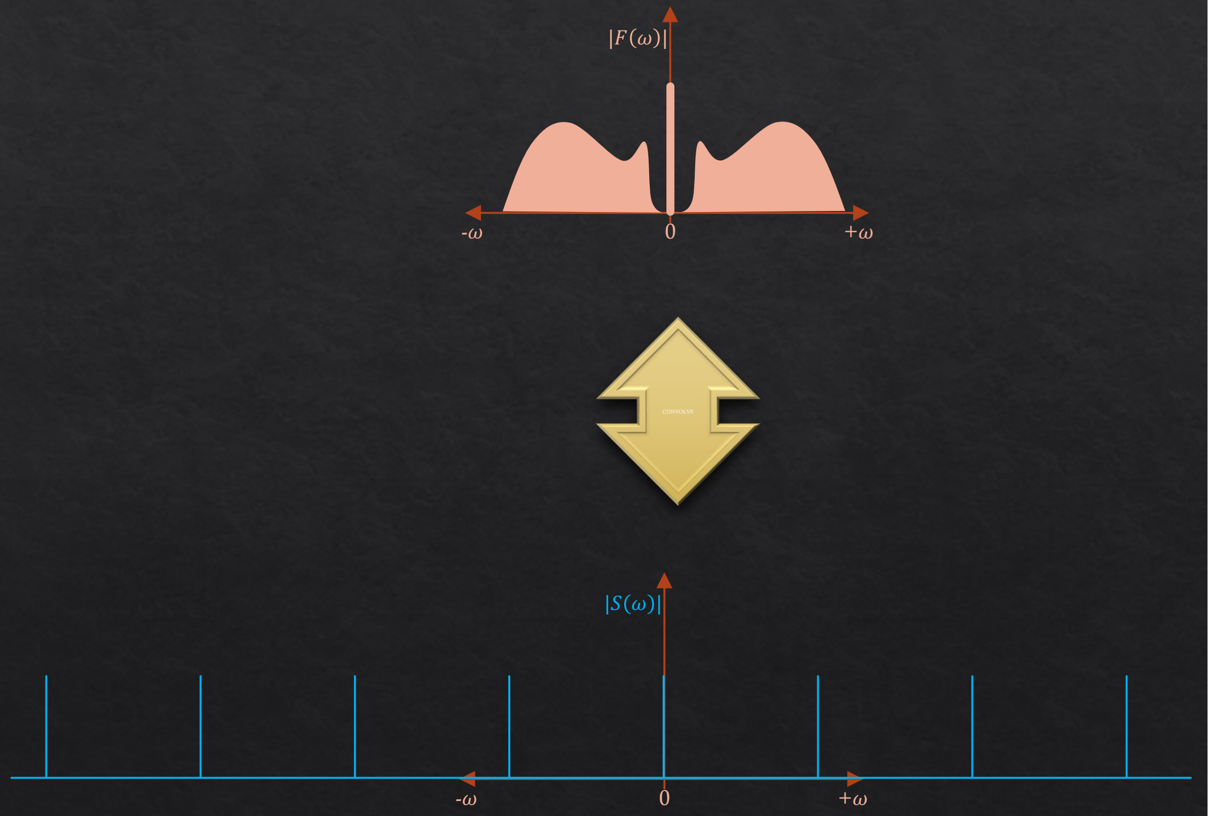




Higher sampling rate = stretched out spectrum

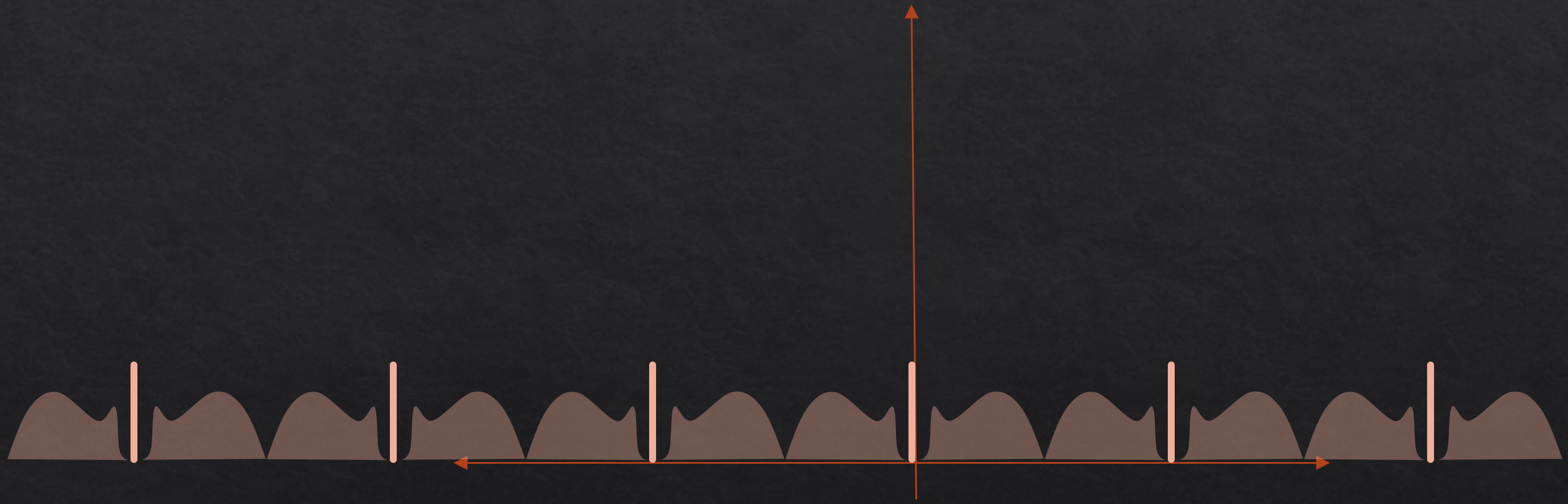
$f_s(x) = f(x) \, . \, s(x)$





Removing aliases: 1) Increase sampling rate

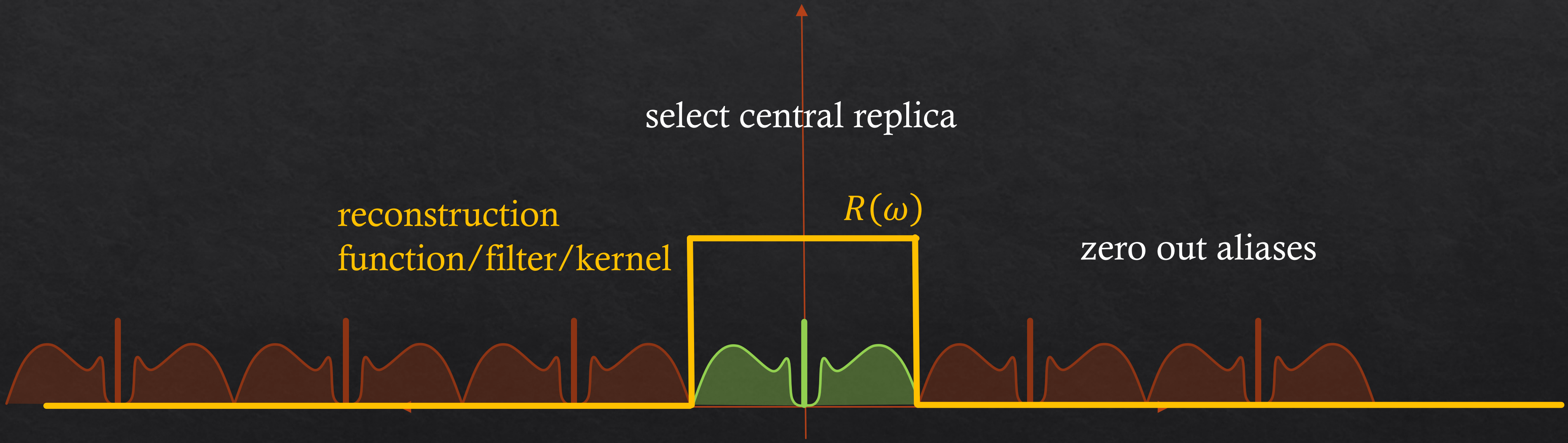
$$F_s(\omega) = F(\omega) \otimes S(\omega)$$



Removing aliases: 2) Crop signal (Fourier)

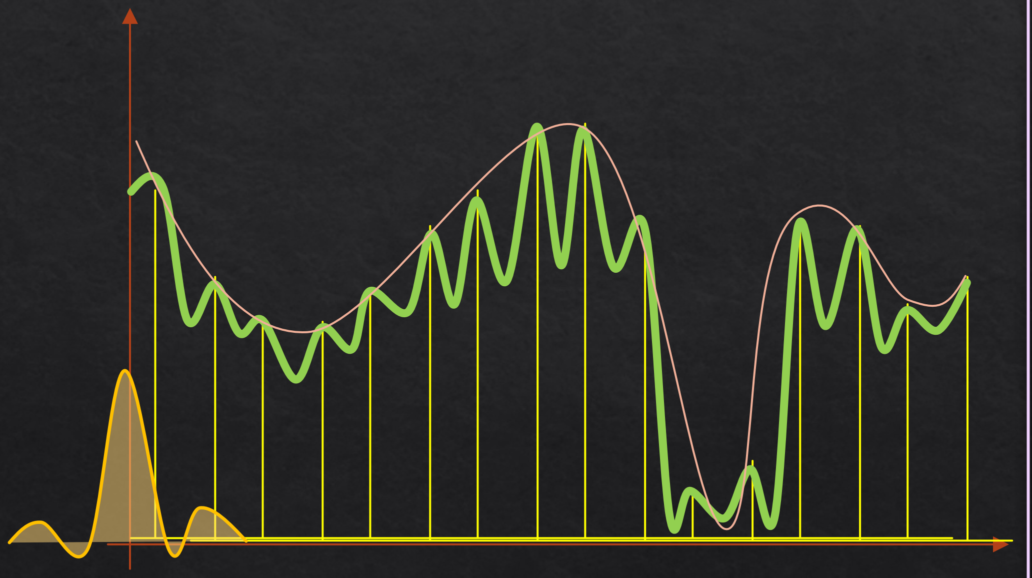
reconstructed signal $F_r(\omega) = R(\omega) \,.\, \big(F(\omega) \otimes S(\omega) \big) = F(\omega)$ original signal if sampling is sufficiently dense

multiply convolve

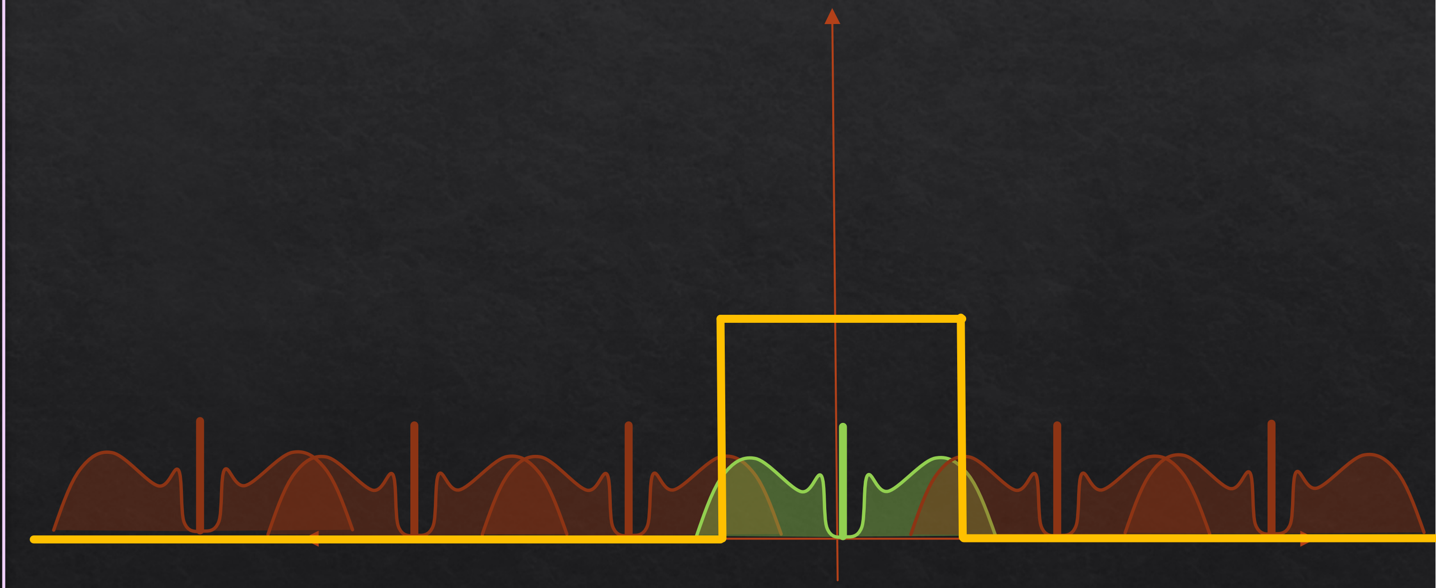


Convolve (Primal) = Crop (Fourier)

$$f_r(x) = r(x) \otimes (f(x) \, . \, s(x))$$



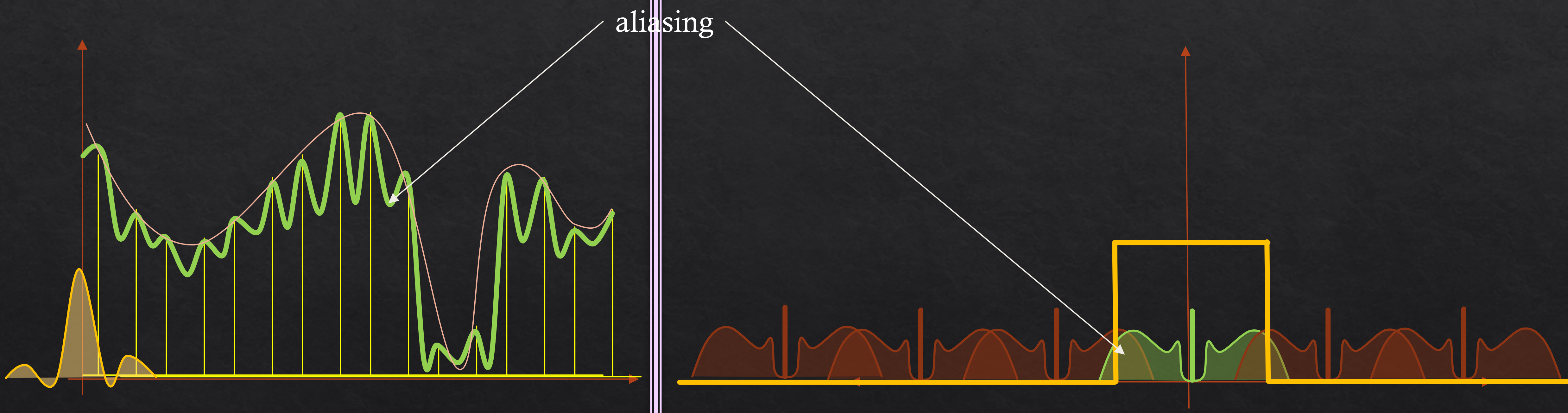
$$F_r(\omega) = R(\omega) \, . \, (F(\omega) \otimes S(\omega)) = F(\omega)$$



Convolve (Primal) = Crop (Fourier)

$$f_r(x) = r(x) \otimes (f(x) \,.\, s(x))$$

$$F_r(\omega) = R(\omega) \,.\, (F(\omega) \otimes S(\omega)) = F(\omega)$$



Convolve (Primal) = Crop (Fourier)

Fourier Transform

$f_r(x) = r(x) \otimes (f(x) \, . \, s(x))$

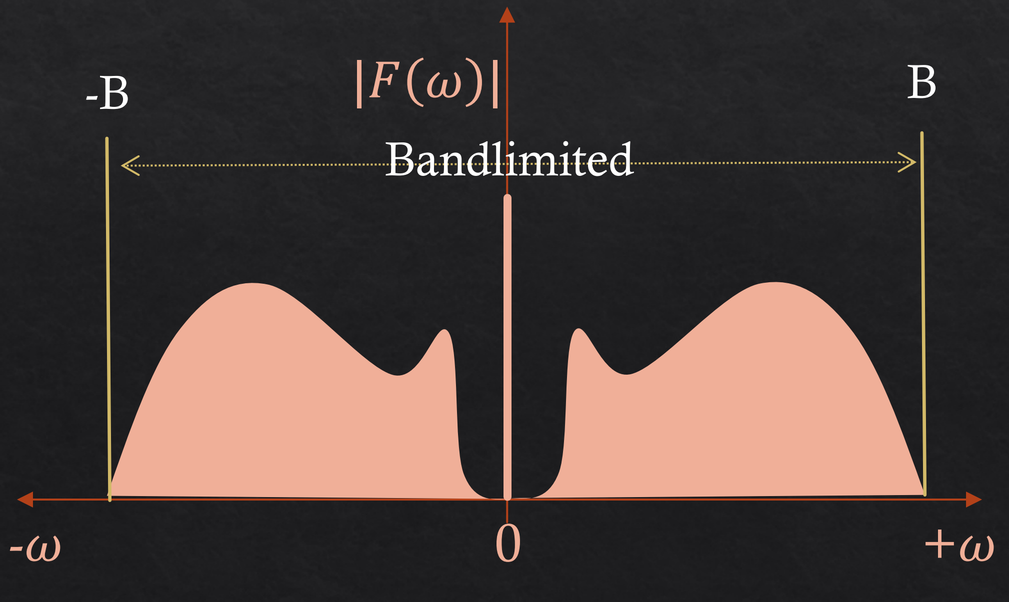
$F_r(\omega) = R(\omega) \, . \, (F(\omega) \otimes S(\omega)) = F(\omega)$

Inverse Fourier Transform

(Nyquist-Shannon) Sampling Theorem

Sampling rate > 2B guarantees no aliasing

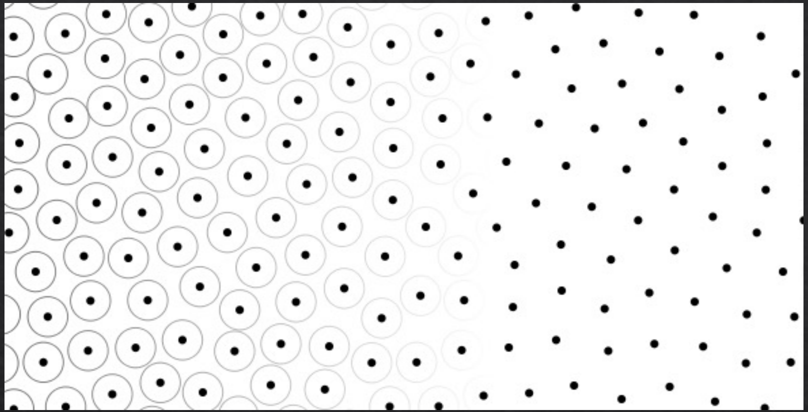
Provided:
1) Function is bandlimited (B is max frequency)
2) Sampling is regular (comb function)

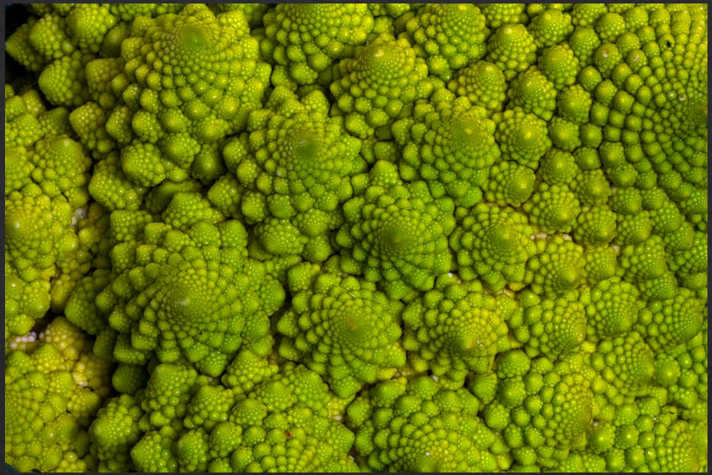


For any dimension: e.g. pixels in 2D

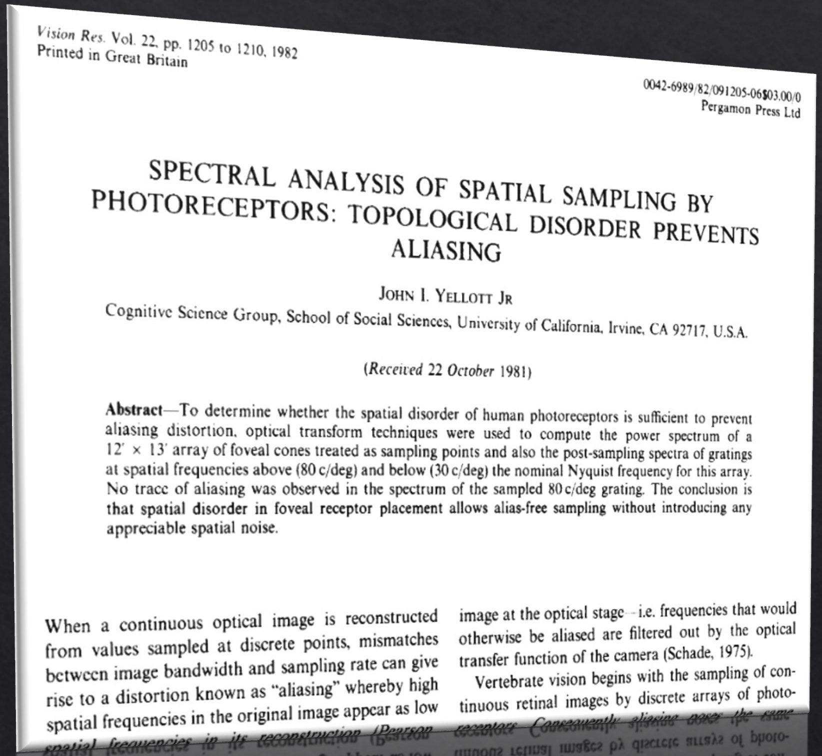
Minimum distance between samples

structure + random





Reconstruction in animals' visual systems



Vision Res. Vol. 22, pp. 1205 to 1210, 1982
Printed in Great Britain

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Pergamon Press Ltd

SPECTRAL ANALYSIS OF SPATIAL SAMPLING BY PHOTORECEPTORS: TOPOLOGICAL DISORDER PREVENTS ALIASING

JOHN I. YELLOTT JR
Cognitive Science Group, School of Social Sciences, University of California, Irvine, CA 92717, U.S.A.

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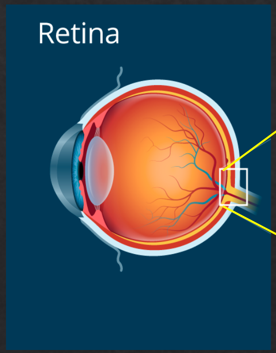
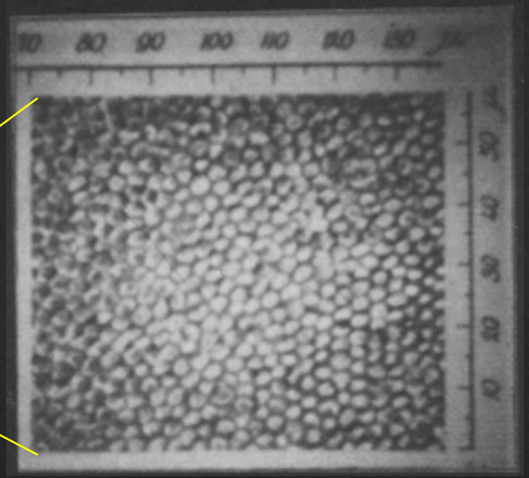
Abstract—To determine whether the spatial disorder of human photoreceptors is sufficient to prevent aliasing distortion, optical transform techniques were used to compute the power spectrum of a 12′ × 13′ array of foveal cones treated as sampling points and also the post-sampling spectra of gratings at spatial frequencies above (80 c/deg) and below (30 c/deg) the nominal Nyquist frequency for this array. No trace of aliasing was observed in the spectrum of the sampled 80 c/deg grating. The conclusion is that spatial disorder in foveal receptor placement allows alias-free sampling without introducing any appreciable spatial noise.

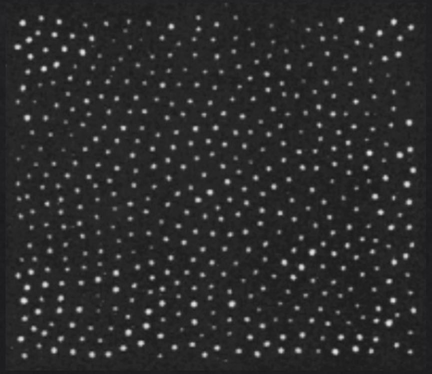
When a continuous optical image is reconstructed from values sampled at discrete points, mismatches between image bandwidth and sampling rate can give rise to a distortion known as "aliasing" whereby high spatial frequencies in the original image appear as low image at the optical stage—i.e. frequencies that would otherwise be aliased are filtered out by the optical transfer function of the camera (Schade, 1975).

Vertebrate vision begins with the sampling of continuous retinal images by discrete arrays of photo-

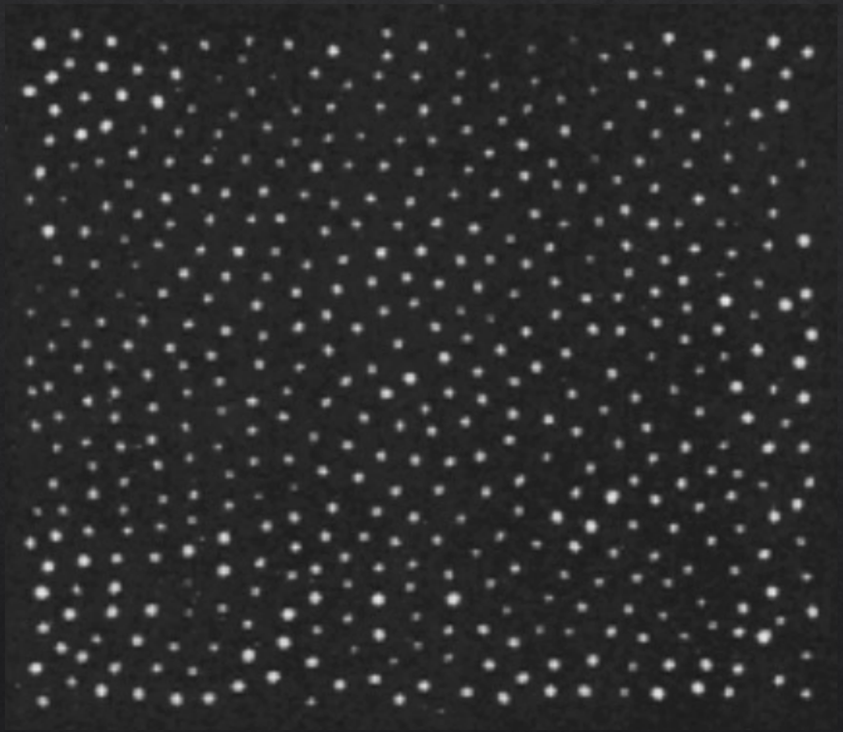




regular structure but not a grid



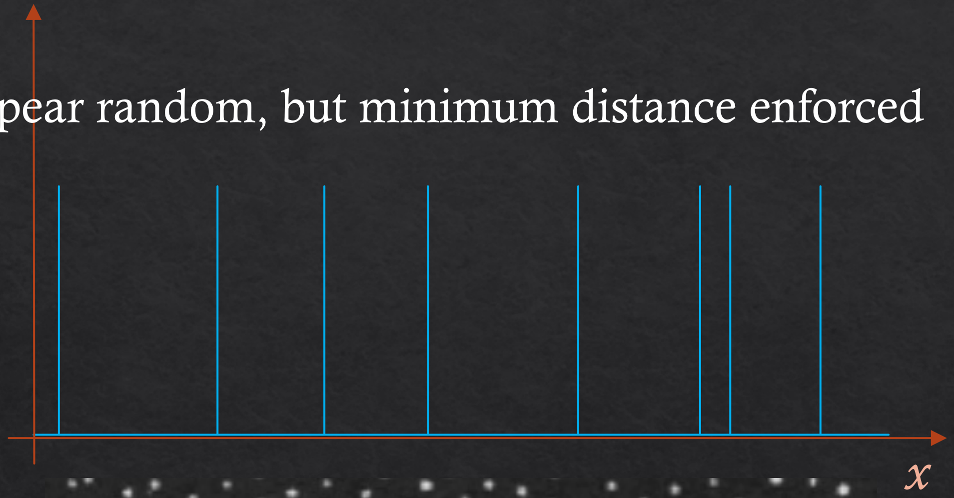
Reconstruction in animals' visual systems

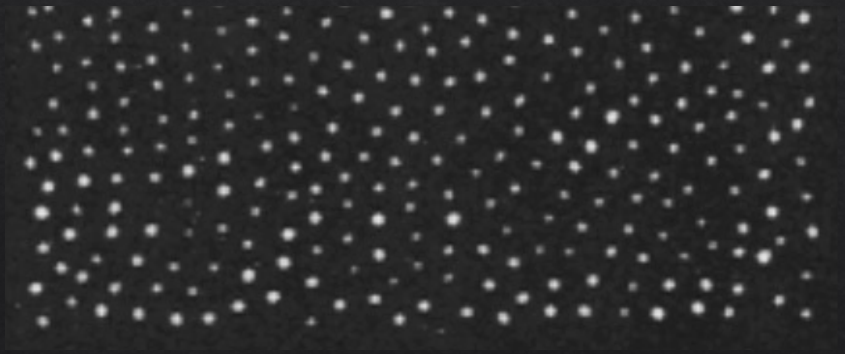


Gap in low-frequencies in Fourier spectrum

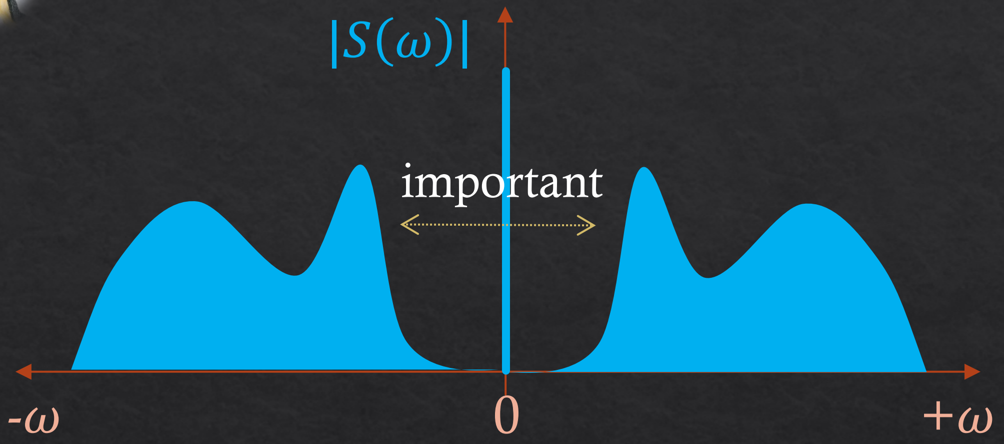
not a comb!

appear random, but minimum distance enforced





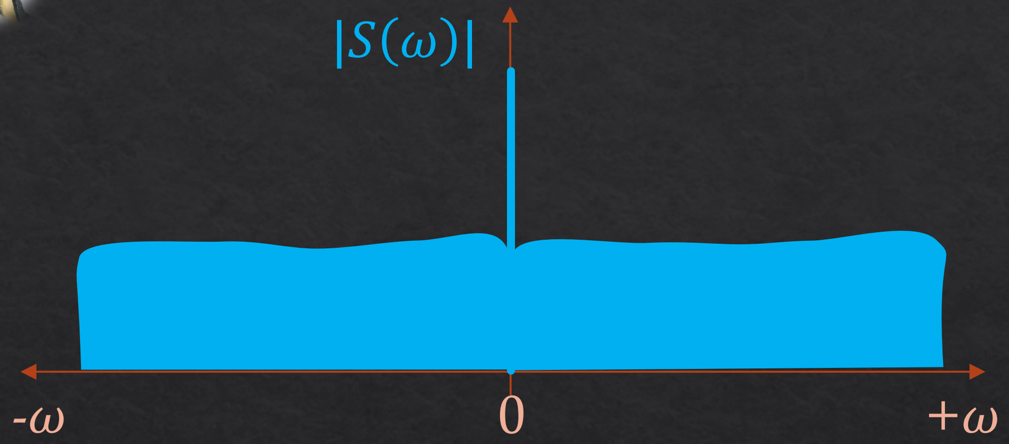
not a comb!



Random sampling spectrum is flat

not a comb!

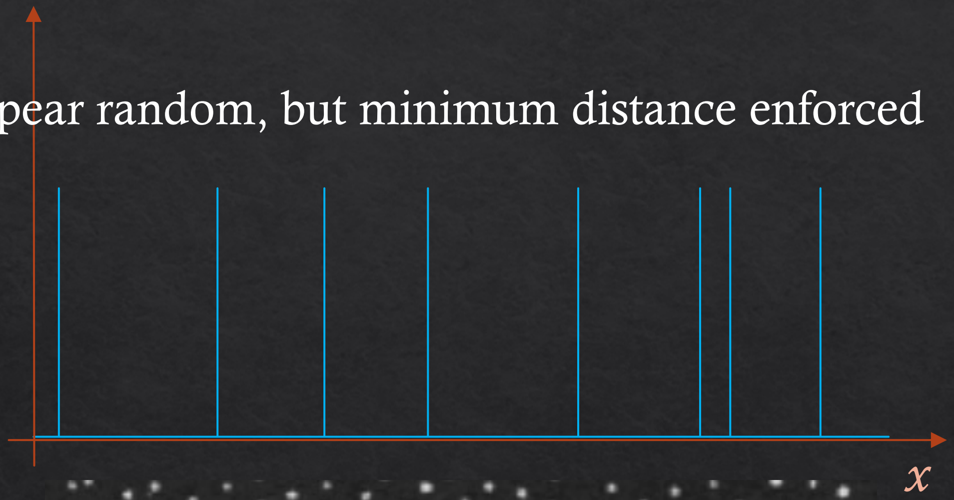
not a comb!

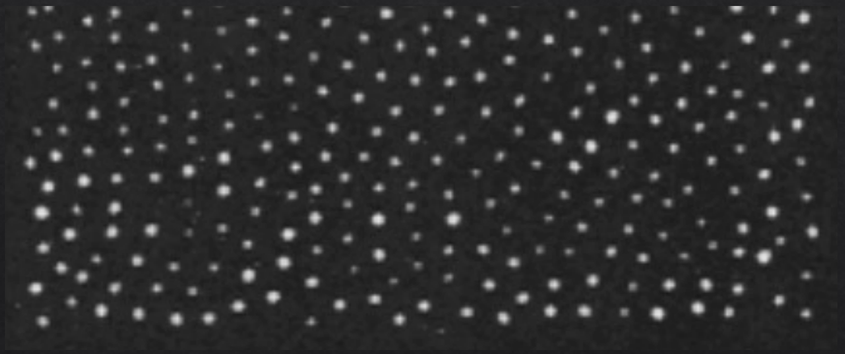


Gap in low-frequencies in Fourier spectrum

not a comb!

appear random, but minimum distance enforced





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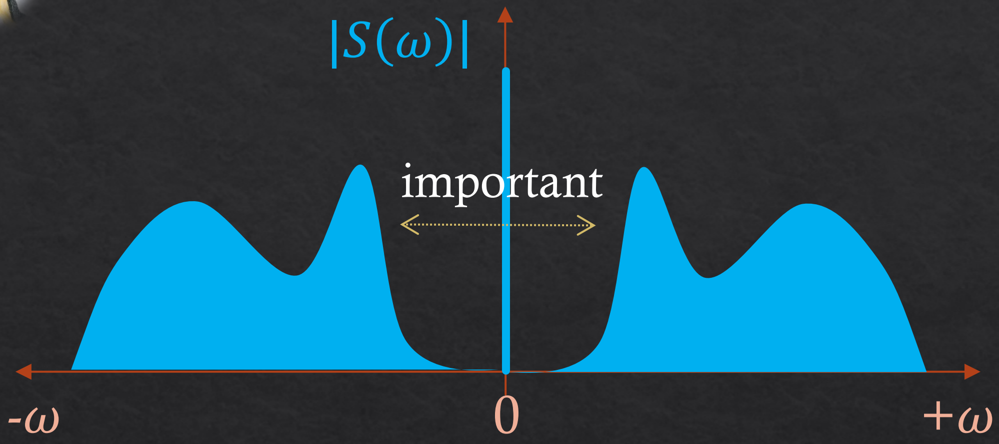
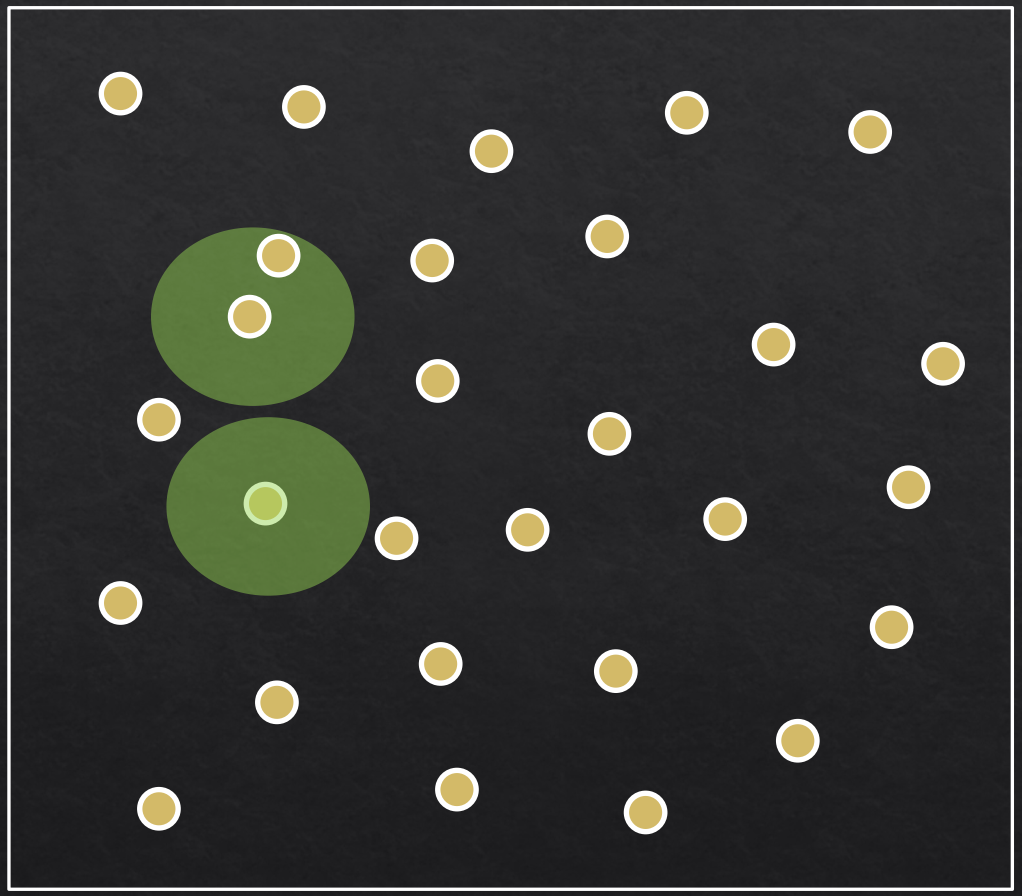


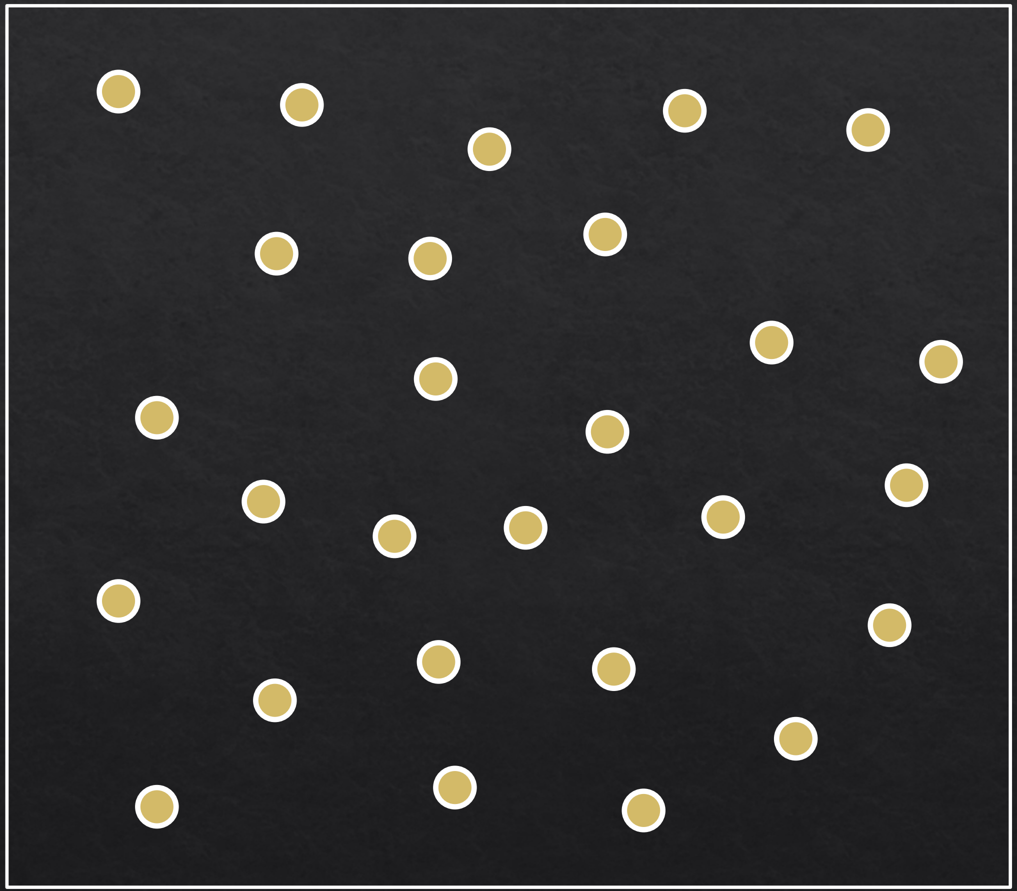
Generating samples: Poisson disk sampling

Generating samples: Poisson disk sampling

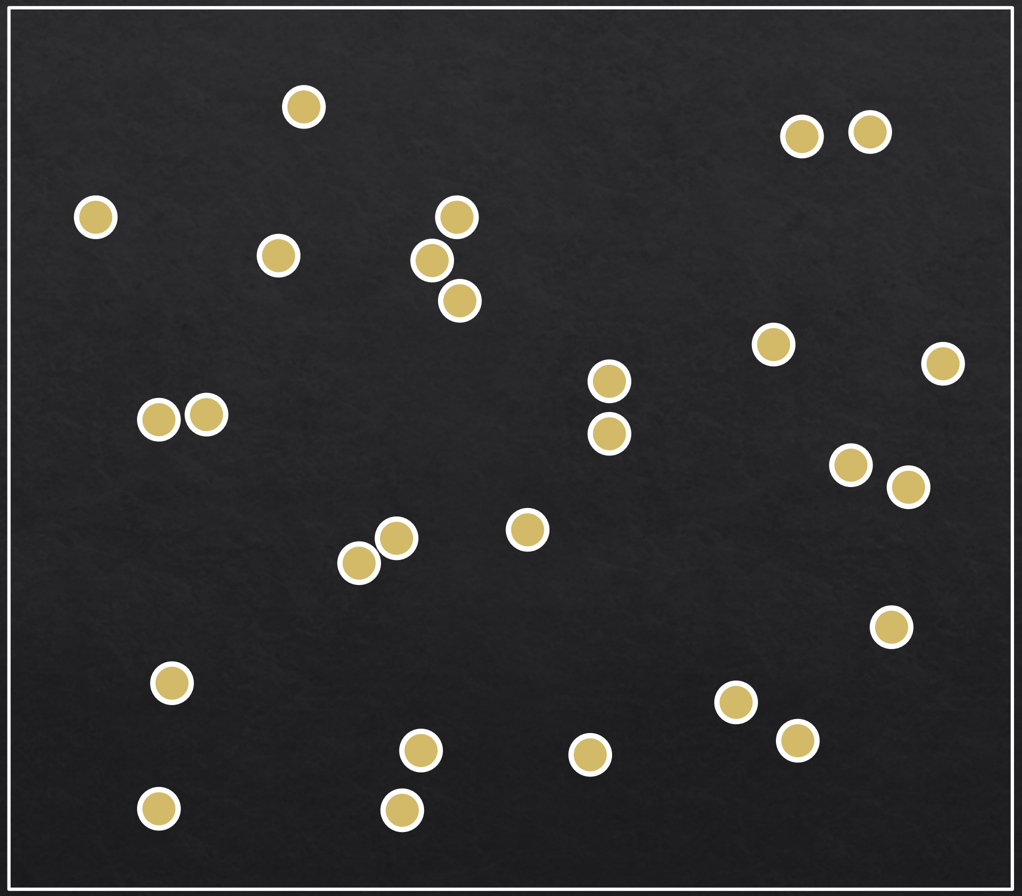
reject sample if closer than minimum distance to any sample



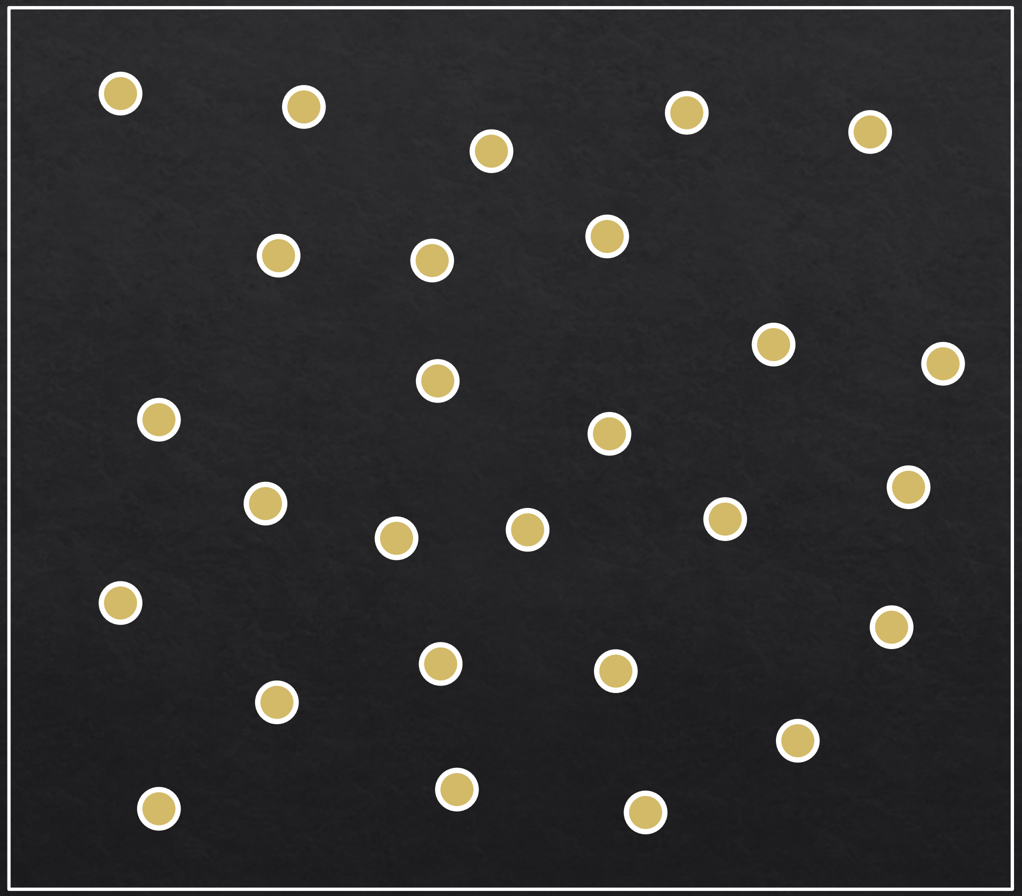
Dart throwing



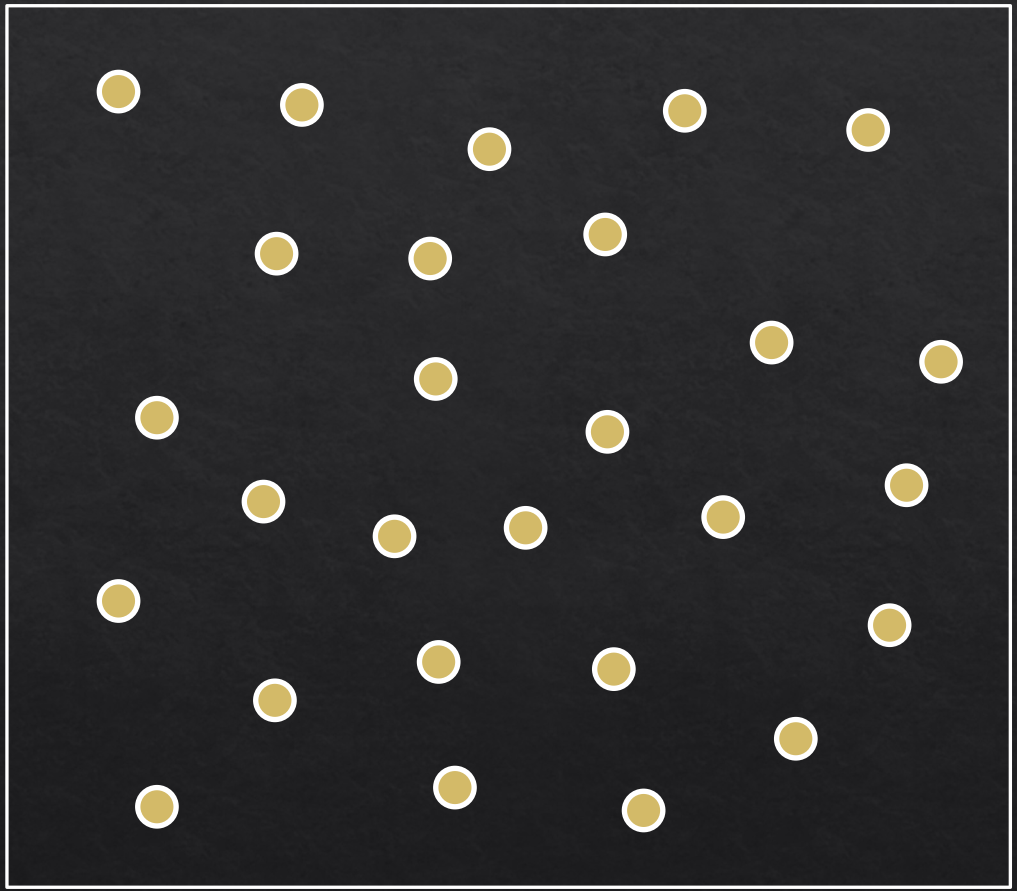
Another approach: start with random samples



Move them until constraint satisfied



Relaxation method



Monte Carlo path tracing ⟶ sampling

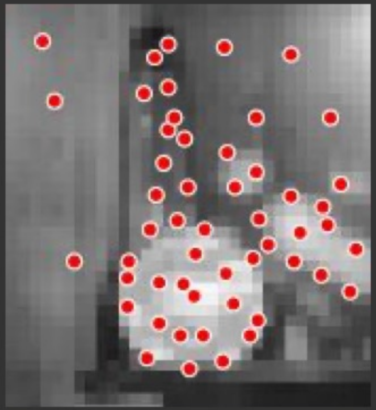
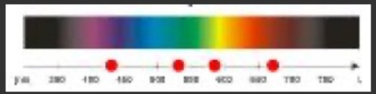


Image space



Visible spectrum

Aperture

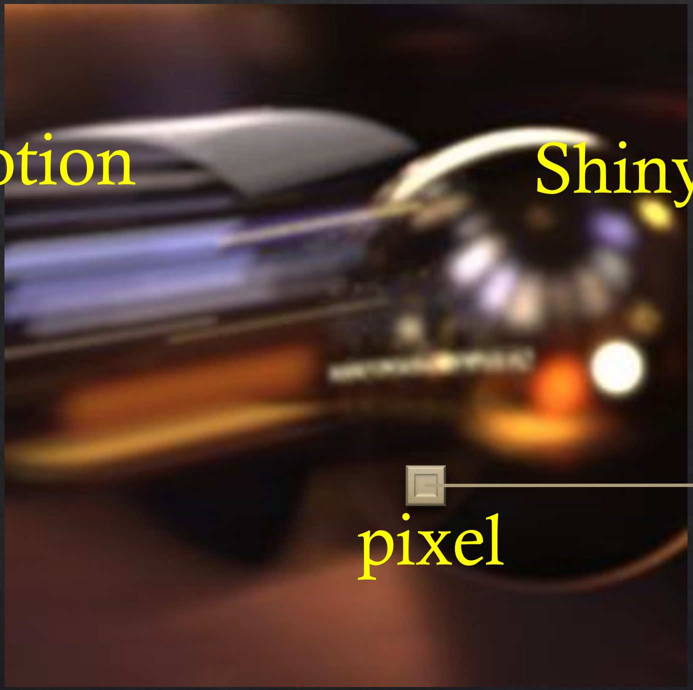
Exposure time

Material reflectance functions

Direct illumination

Indirect illumination

Light transport = integration



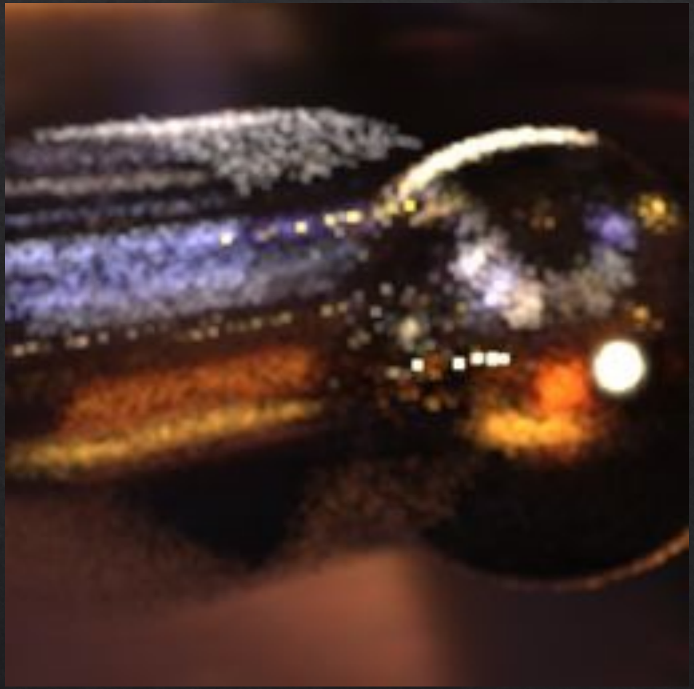
$$\int\int\int\int\cdots$$

multi-dim integral

Integrand: radiance (W m^{-2} Sr^{-1})

Domain: pixel area x shutter time x aperture area x 1^{st} bounce x 2^{nd} bounce…

Variance and bias

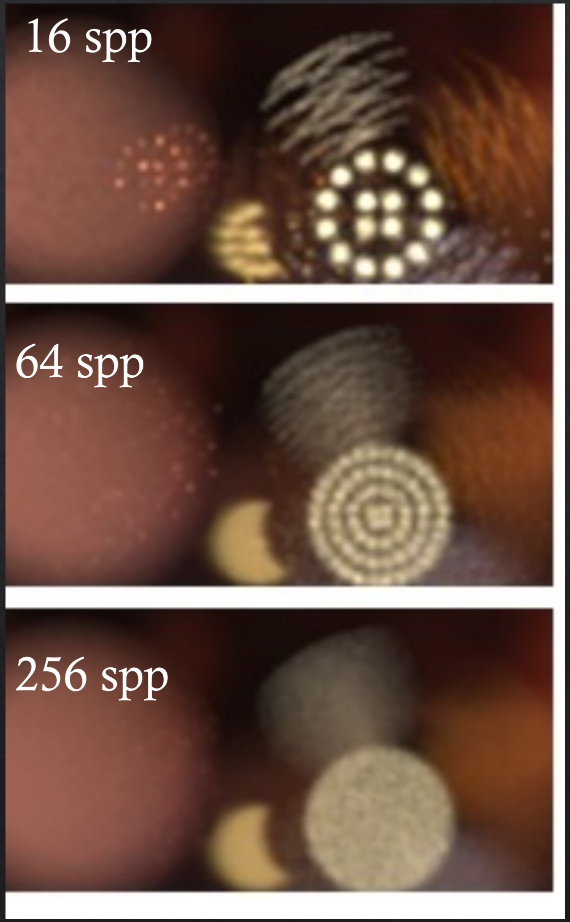


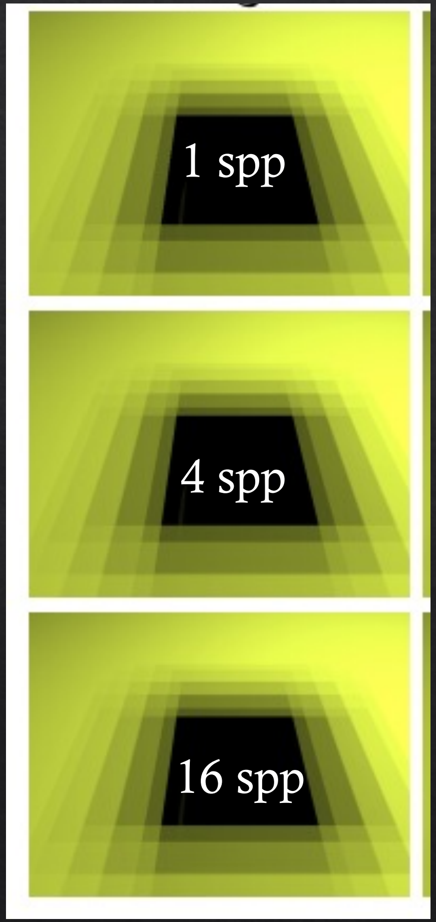
High variance

High bias

For any dimension: e.g. light paths > 2D

comb (regular grid)





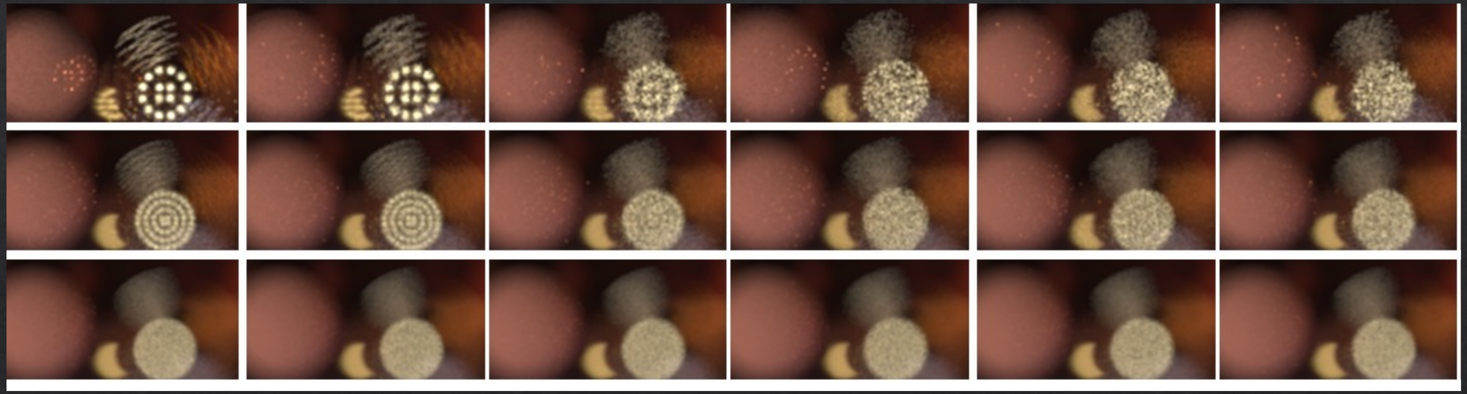
But this is numerical integration, not reconstruction !

What is the connection between these two classes of problems?

Adding randomness is good. Why?

comb (regular grid) → jittered grid

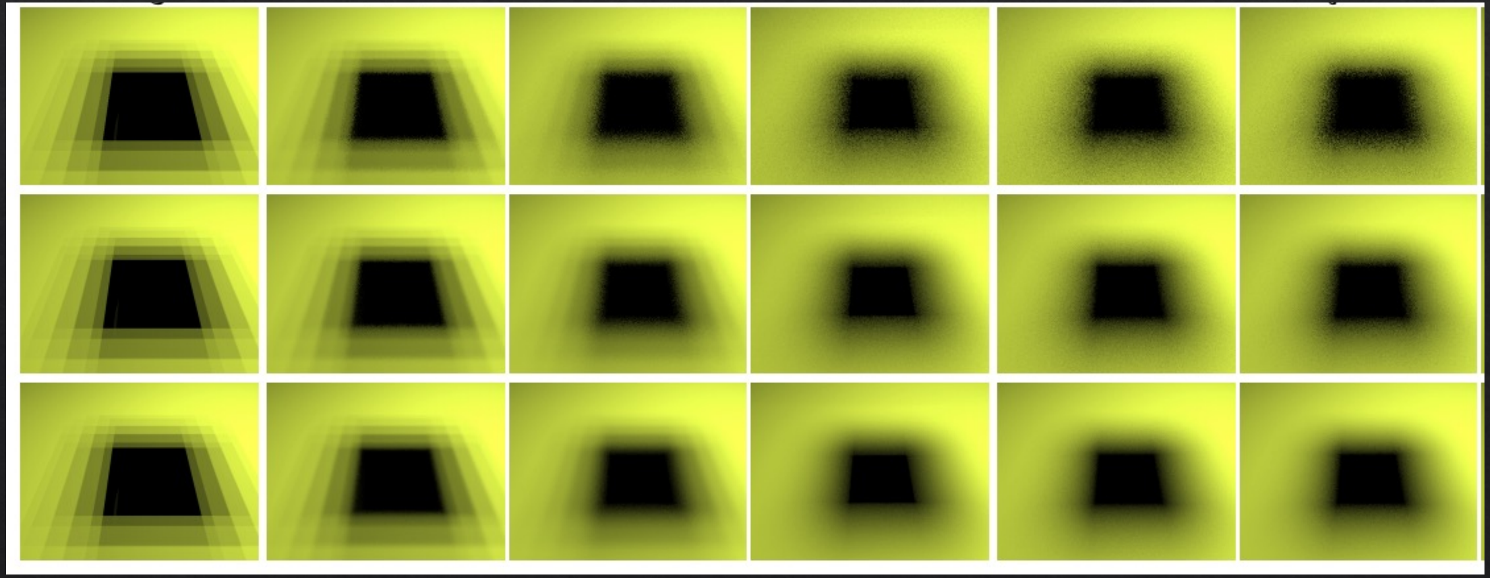
16 spp



structured artifacts are visually disturbing

random noise is less objectionable although undesirable

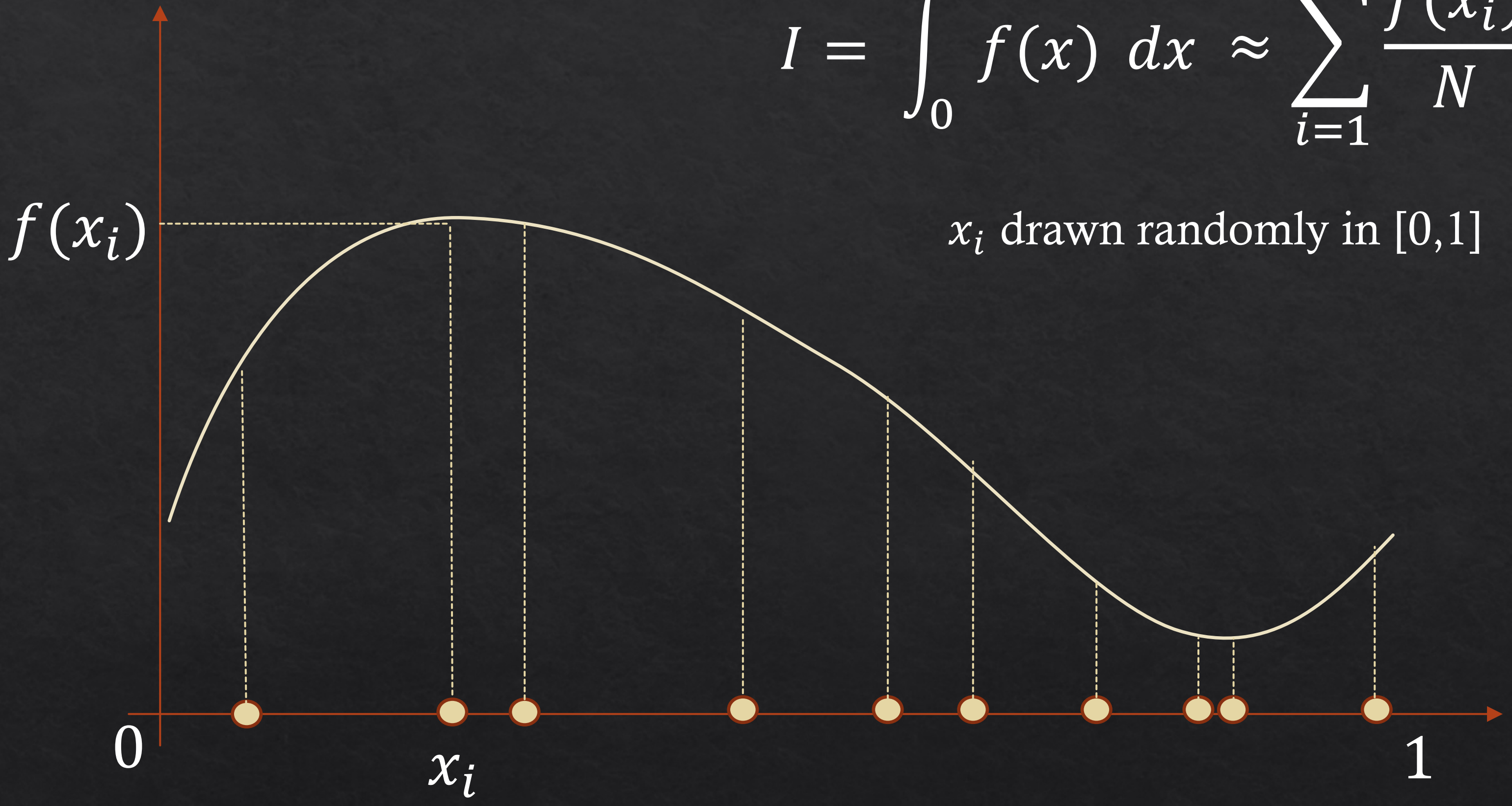
1 spp



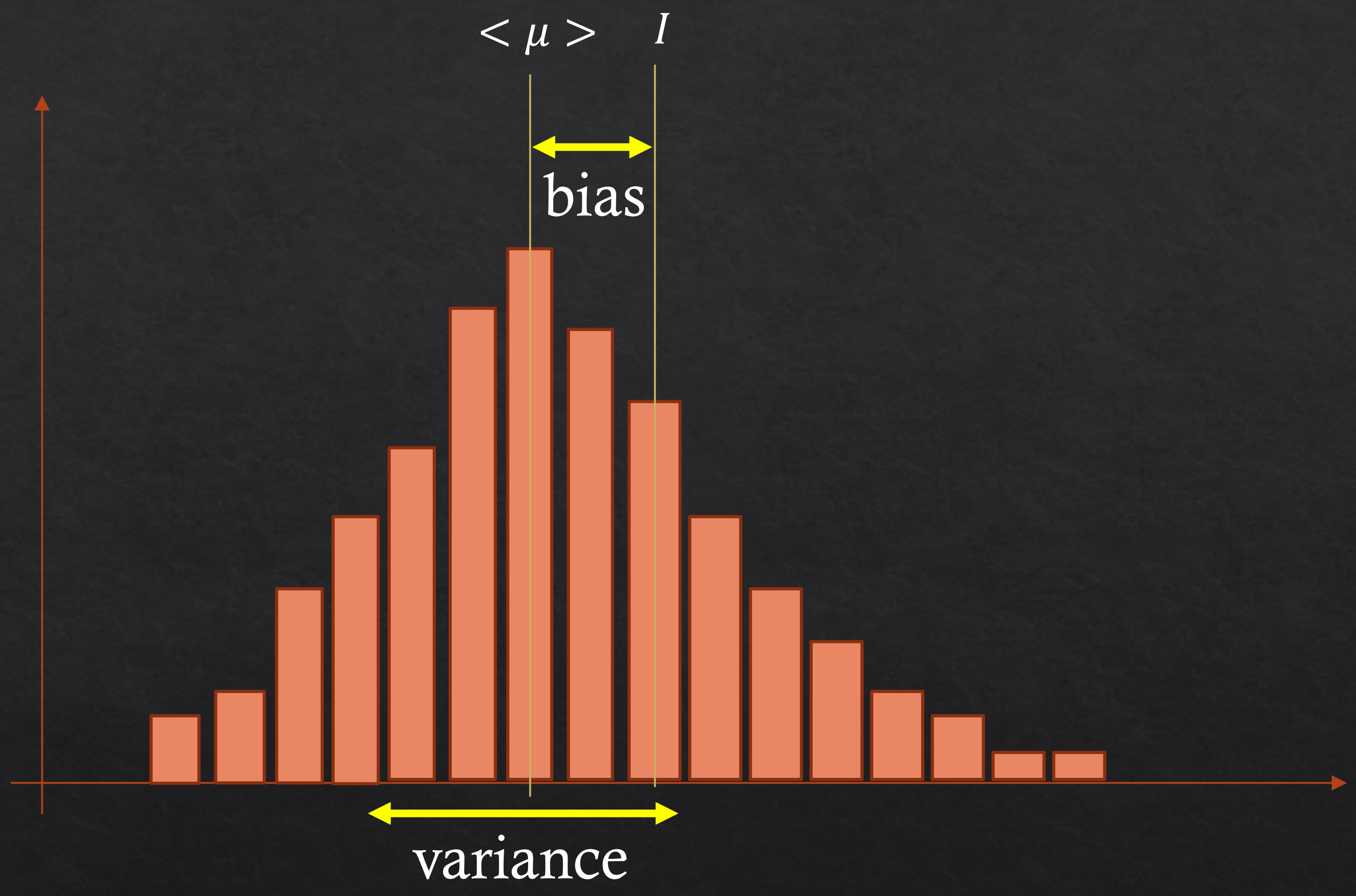
Monte Carlo integration is an approximation

$$I = \int_0^1 f(x)\ dx \approx \sum_{i=1}^{N} \frac{f(x_i)}{N}$$

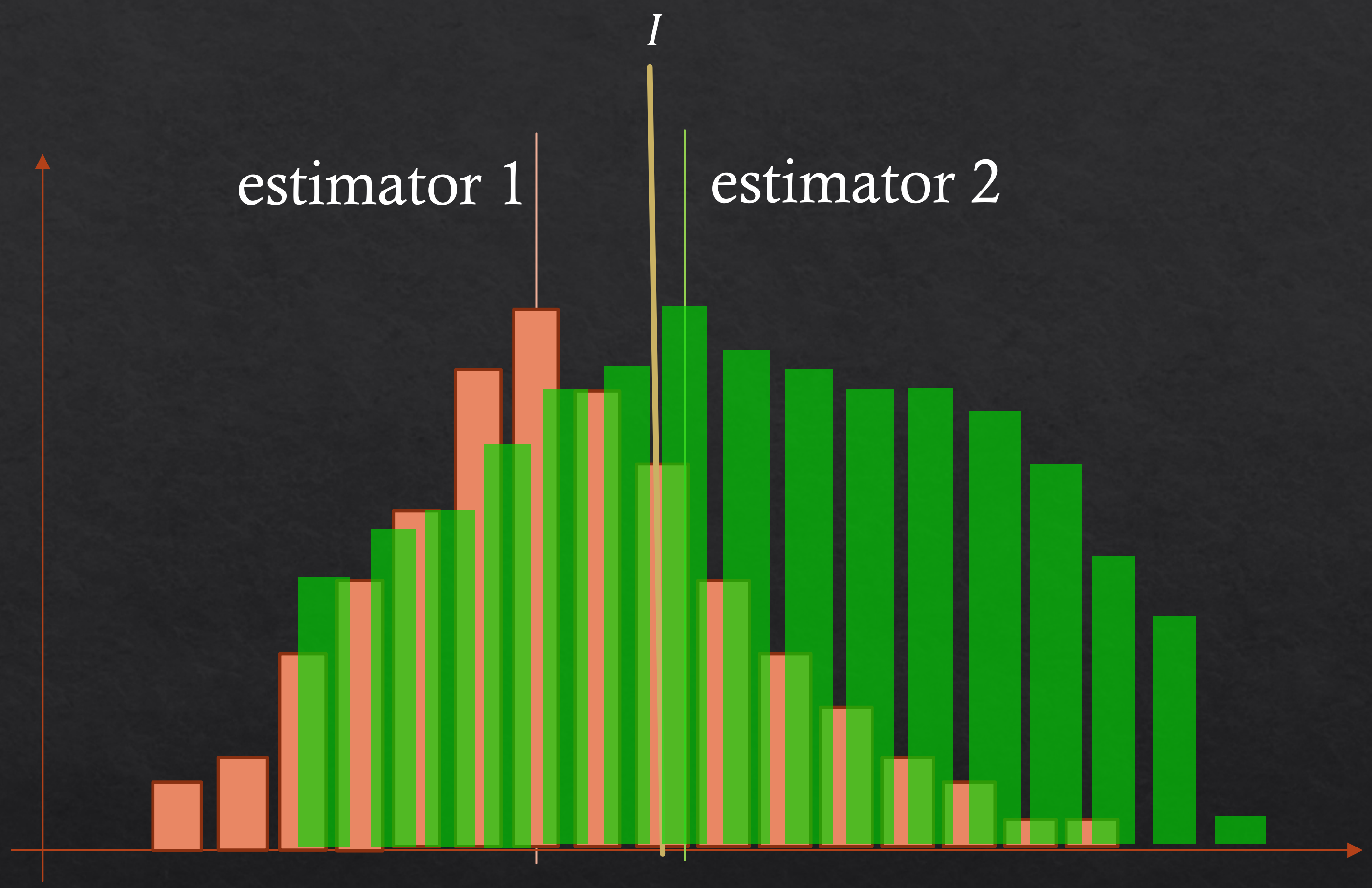
x_i drawn randomly in [0,1]



Error due to sampling: histogram of estimates



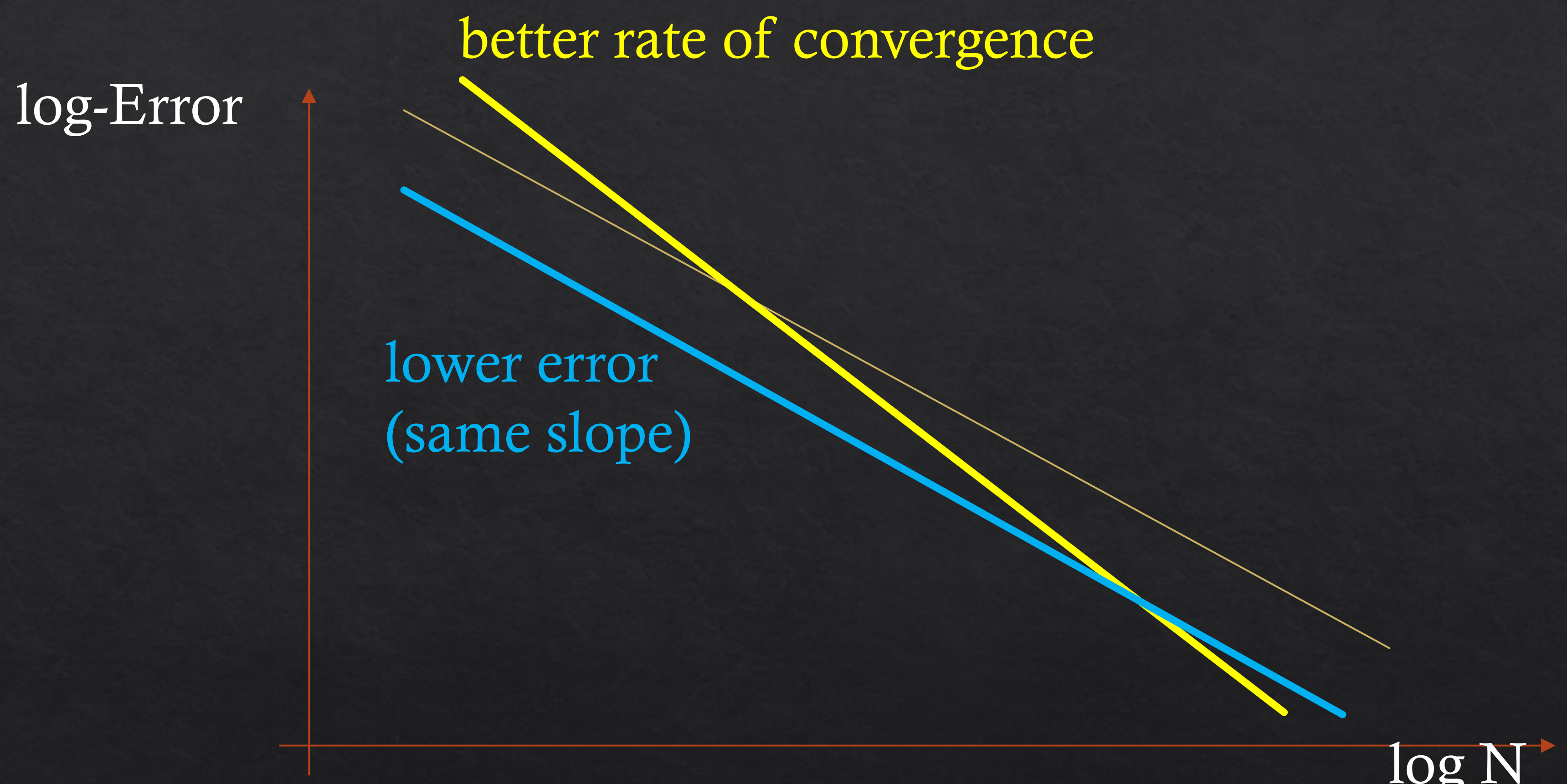
Which estimator is better?



Convergence as N is increased

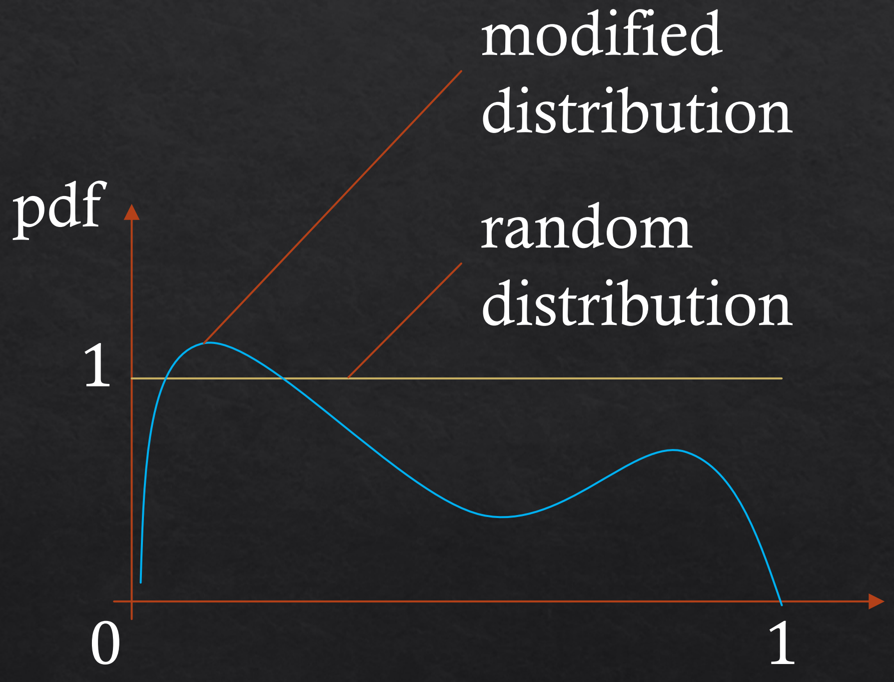
Monte Carlo convergence: $O(1/\sqrt{N})$
slope = -1/2

Two classes of improvements



But how?

change sampling distribution



introduce sample correlations
(e.g. using a grid-structure)