Computer Graphics

Lecture 13: Path tracing

Kartic Subr

THE UNIVERSITY of EDINBURGH informatics

1 Go to wooclap.com

2 Enter the event code in the top banner

Event code
OHECTR

1 Send @OHECTR to (0113) 320 9662

2 You can participate

I spent too long on CW1…

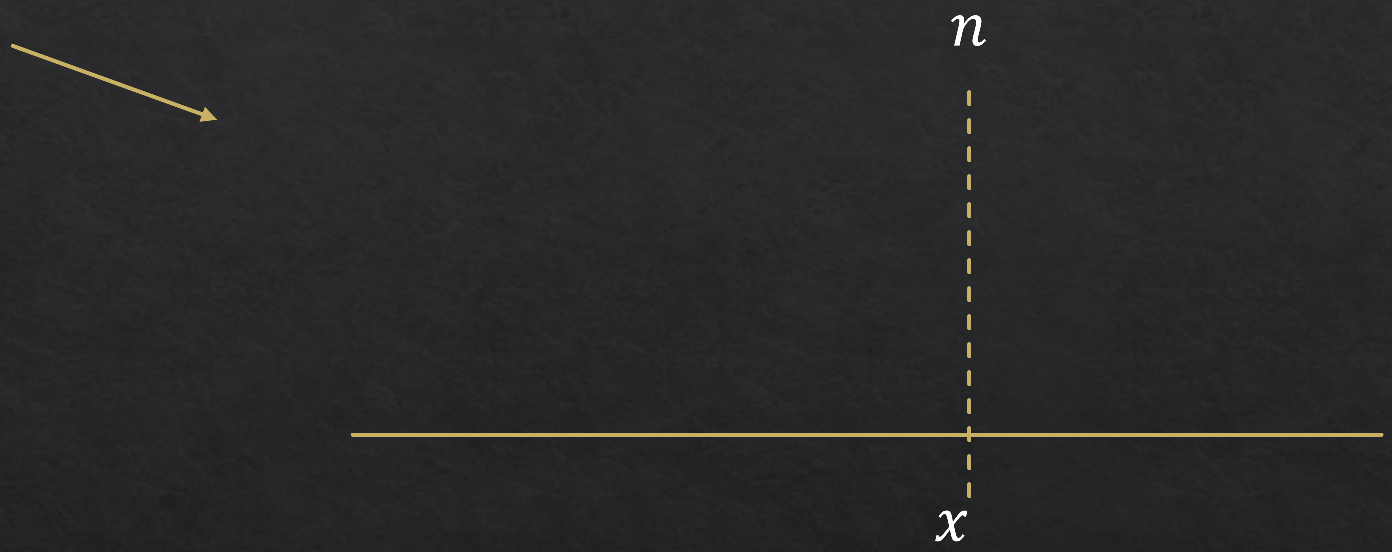
- I spent too much time on little details

- I should have hacked it together

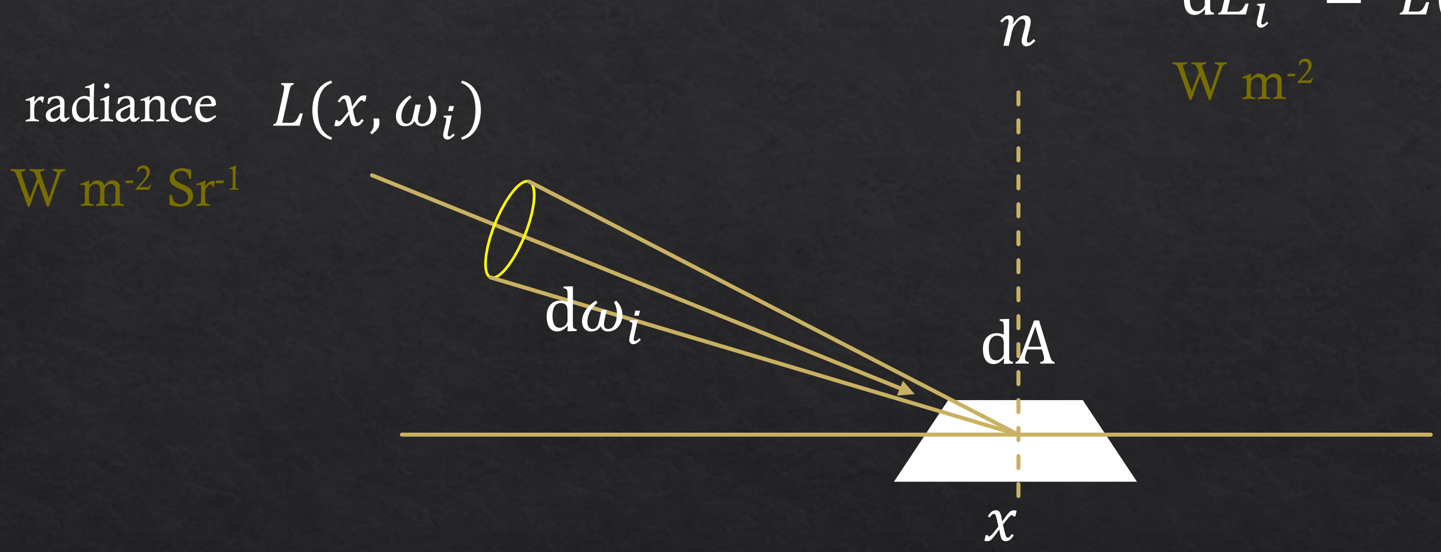
- I should have focused on completing easy tasks from the marking scheme

- I should have started earlier

Incident or incoming radiance at x



Differential irradiance at infinitesimal patch

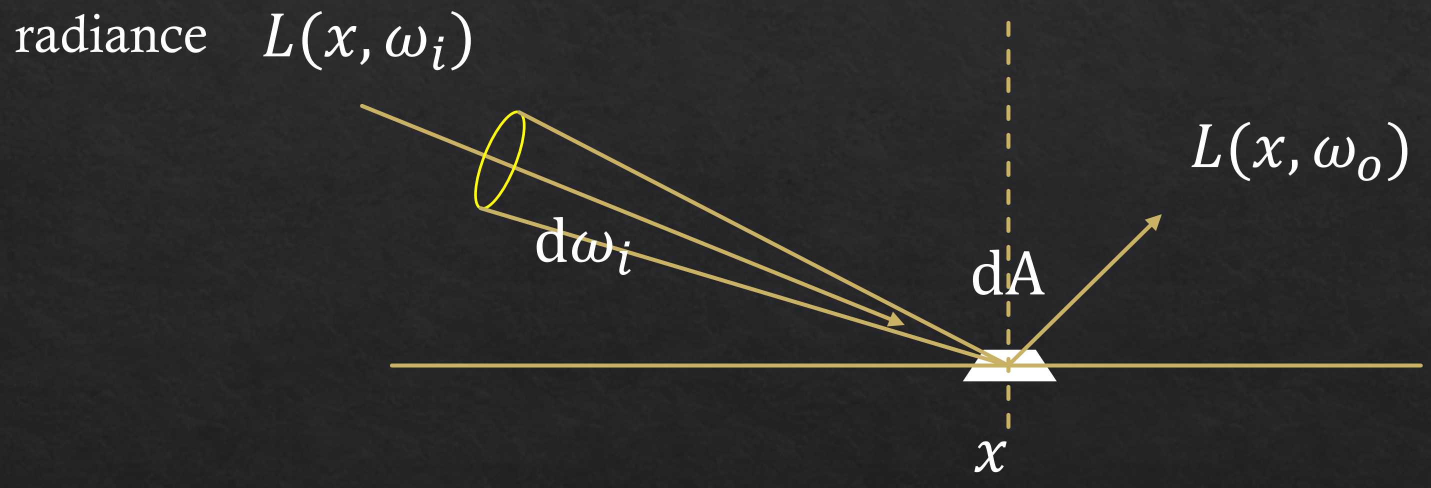


$$dE_i \;=\; L(x, \omega_i)\,(\omega_i.\mathrm{n})\,d\omega_i$$

W m-2

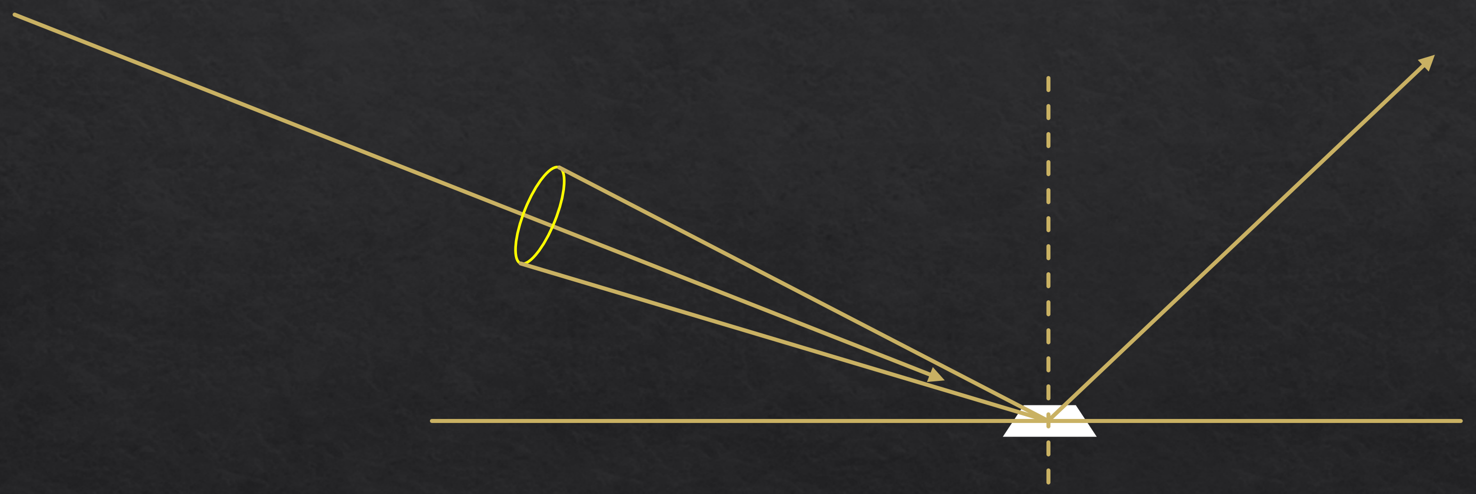
differential irradiance at x, due to one incident ray

Outgoing radiance along a direction



Linear optics: reflected radiance ∝ irradiance

$$\mathrm{d}L(x,\omega_o) = \rho(\omega_i,\omega_o)\ \ \mathrm{d}E_i$$



Constant of proportionality is a function!

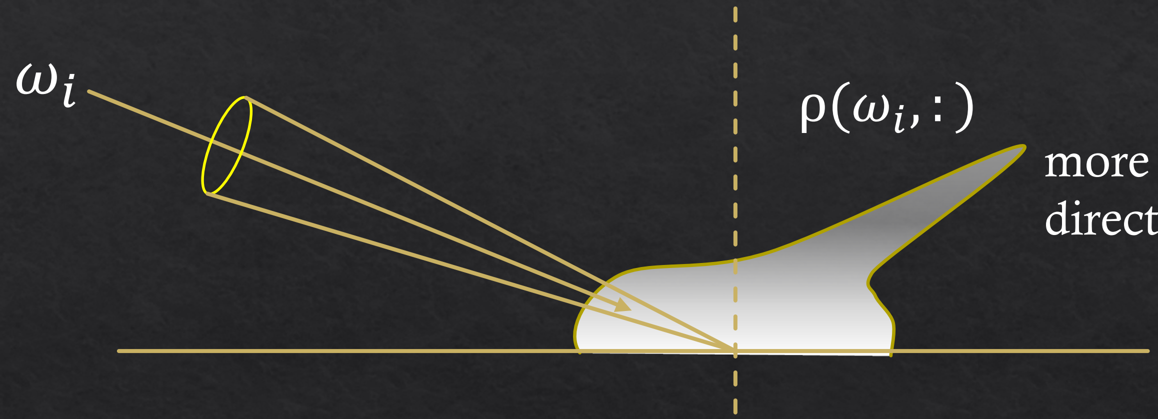
$$\mathrm{d}L(x, \omega_o) = \rho(\omega_i, \omega_o) \quad \mathrm{d}E_i$$

Constant for a given pair of incident-outgoing directions

Determines appearance of opaque materials

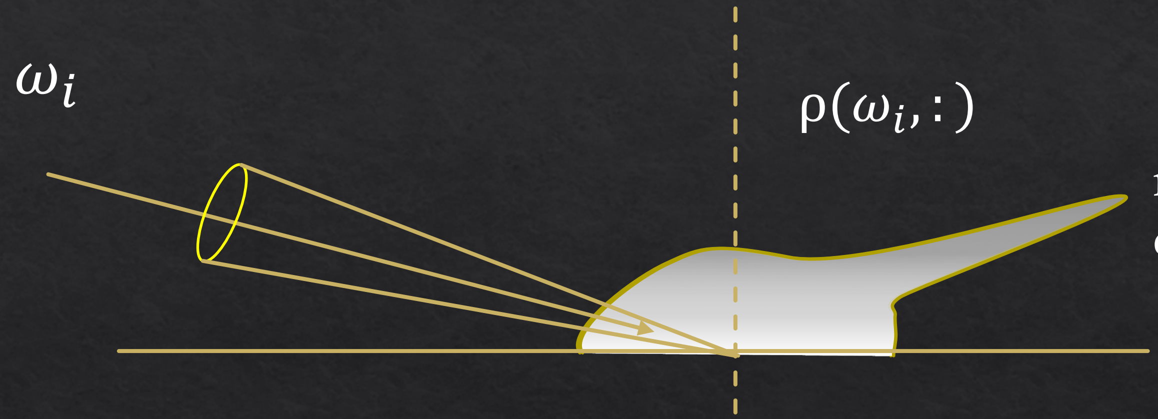
Bidirectional Reflectance Distribution Function (BRDF)-

BRDF slice per incident direction



more light along
direction of specular reflection

BRDF slice per incident direction



more light along
direction of specular reflection

BRDF measurement - gonioreflectometer

tabulate 4D measured values?

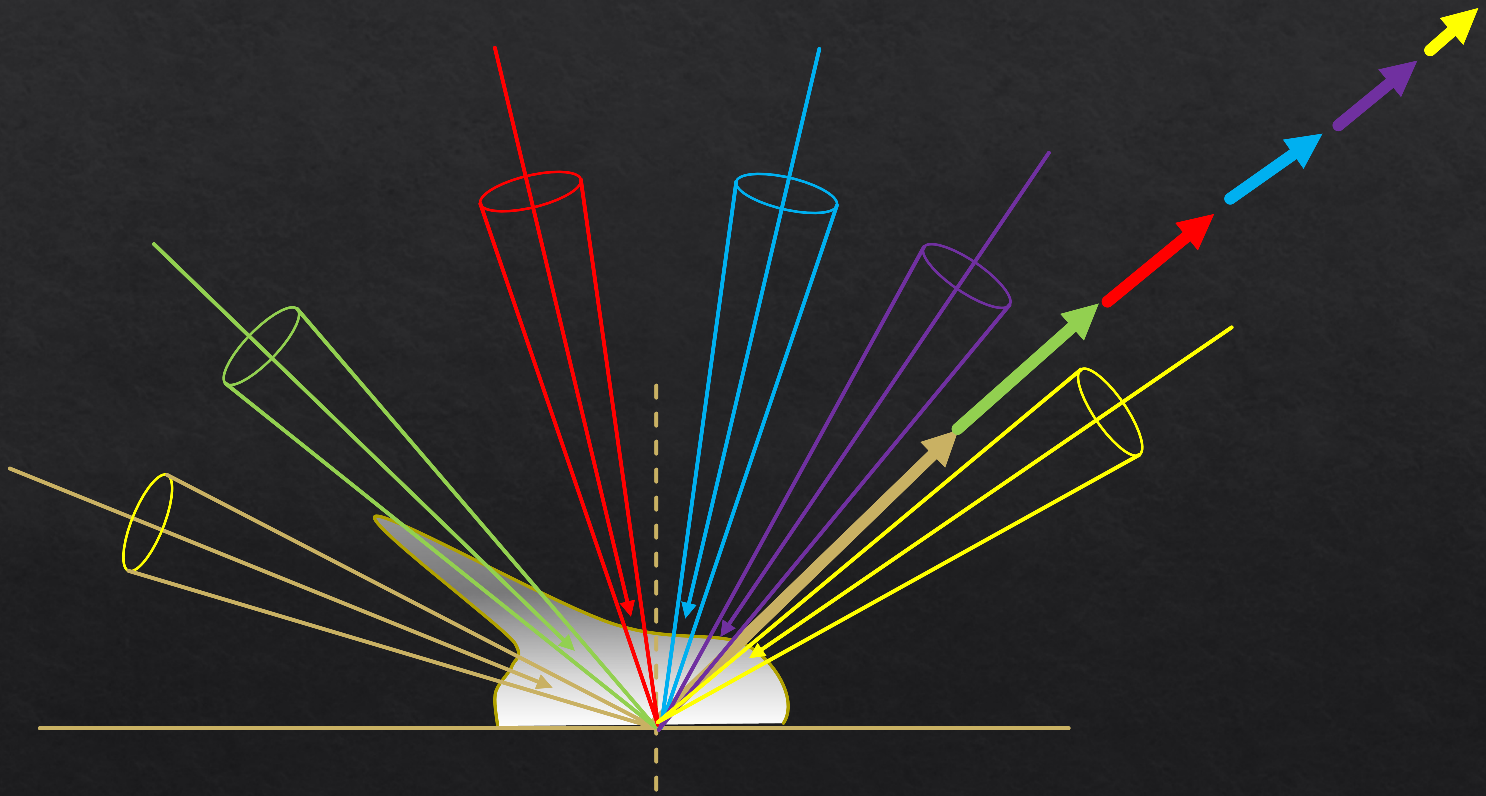
[robotae]

http://www.cs.cmu.edu/afs/cs/academic/class/15462-f09/www/lec/lec9.pdf

commonly used material models

Reflection: multiple incident rays

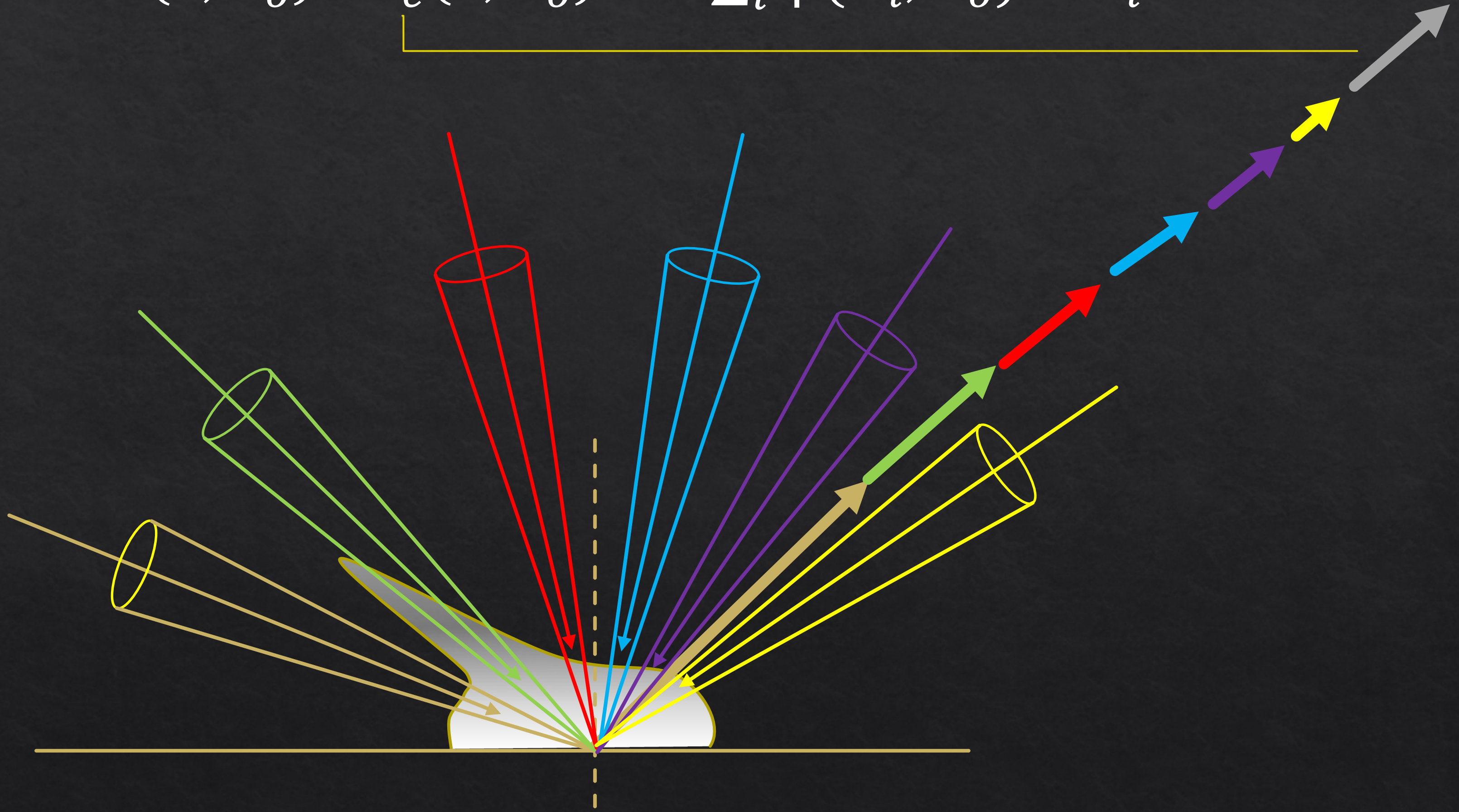
$$L(x, \omega_o) = \sum_i \rho(\omega_i, \omega_o) \ \ dE_i$$



contributions for different directions weighted by BRDF

Add emission from surface at x

$$L(x,\omega_o) = L_e(x,\omega_o) + \sum_i \rho(\omega_i,\omega_o)\ \mathrm{d}E_i$$



contributions for different directions weighted by BRDF

In the limit …

$$L(x,\omega_o) = L_e(x,\omega_o). + \int_{H^2} \rho(\omega_i,\omega_o)\ L(x,\omega_i)\ (\omega_i.\mathrm{n})\ \mathrm{d}\omega_i$$

surface emission

hemisphere

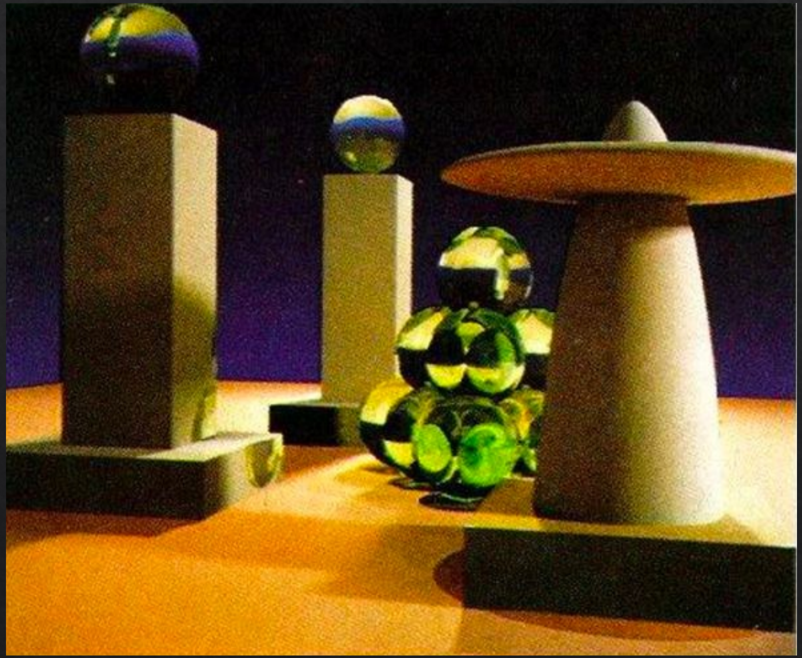
BRDF

incident radiance

cosine dependence

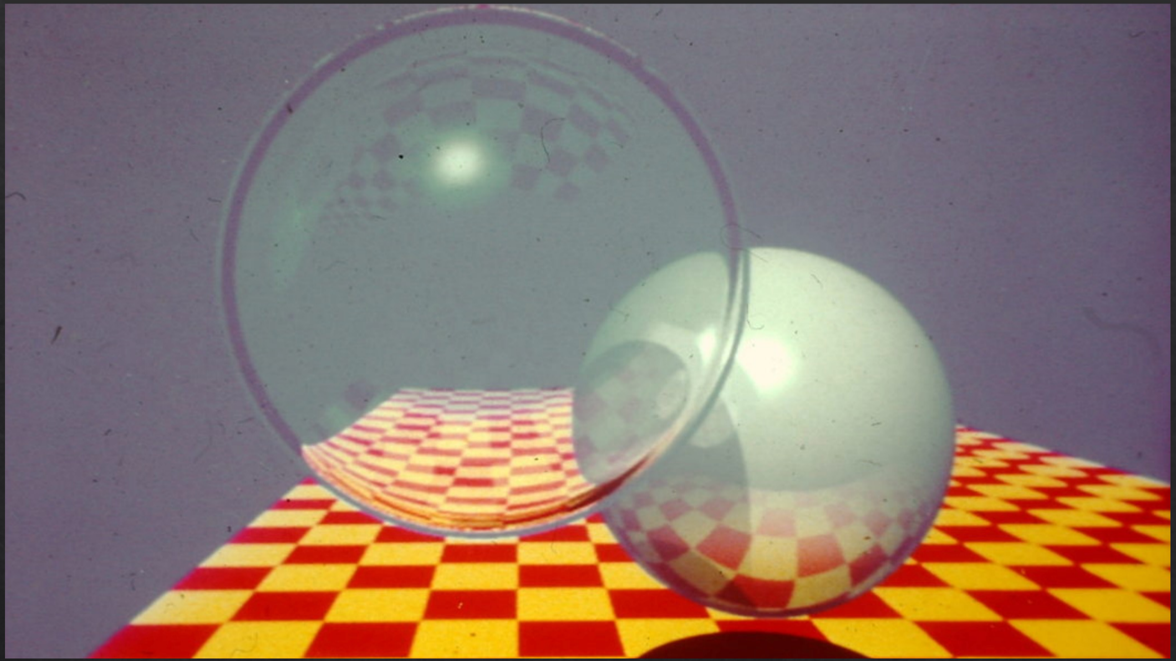
… the rendering equation [Kajiya 86]

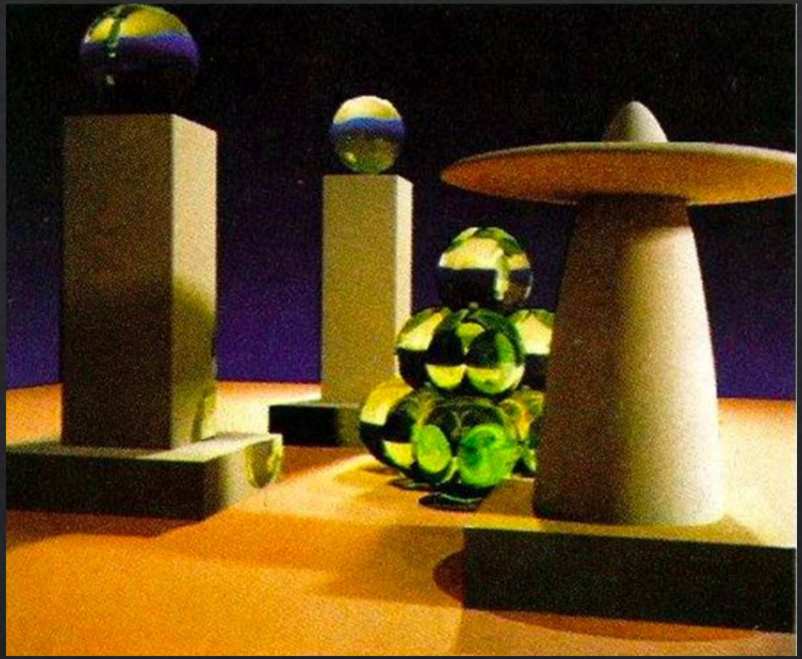
The rendering equation



[Kajiya 86] https://dl.acm.org/citation.cfm?id=15902

Contrast with Whitted raytracing





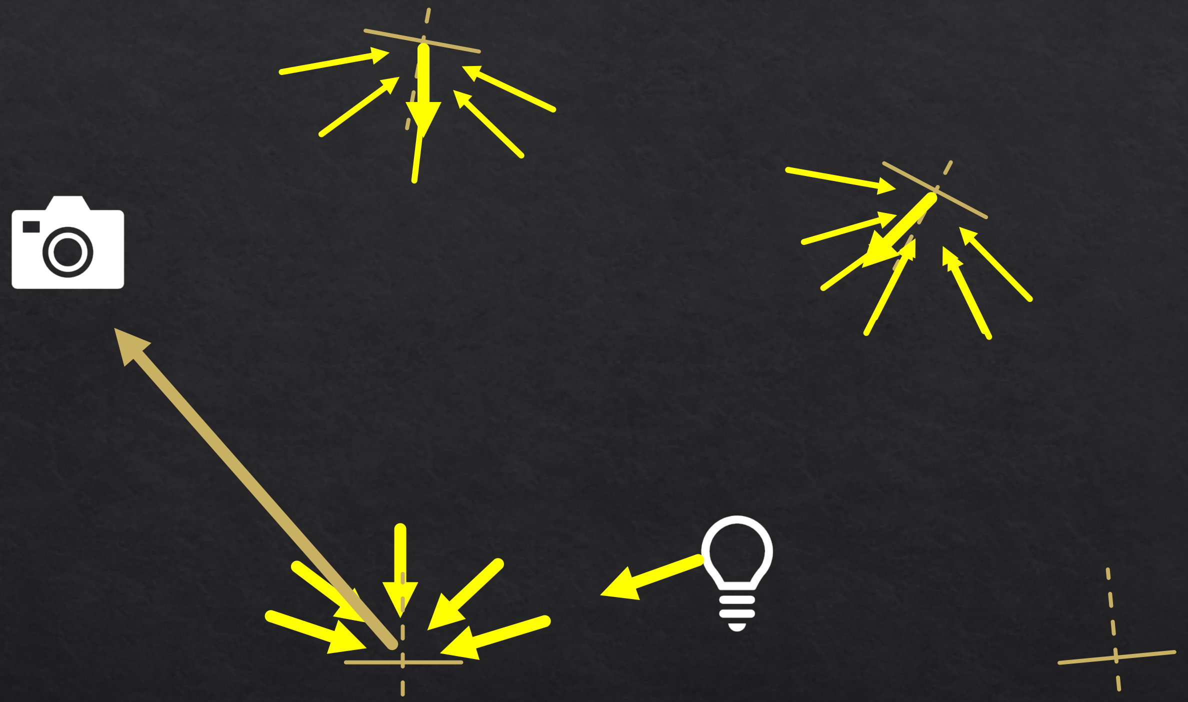
How to get ‘soft’ shading and lighting effects?

Solving the rendering equation

Estimate integrals recursively

1) Sample hemisphere at last bounce to camera

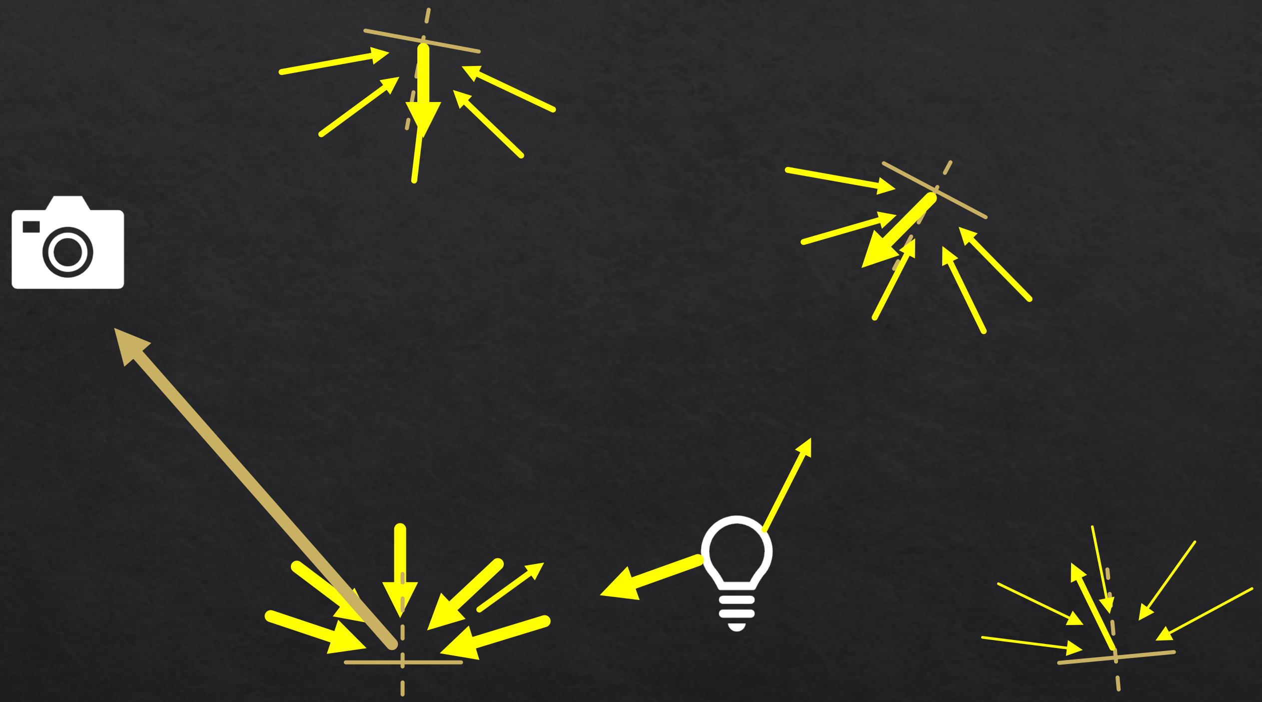
Estimate integrals recursively



1) Sample hemisphere at last bounce to camera

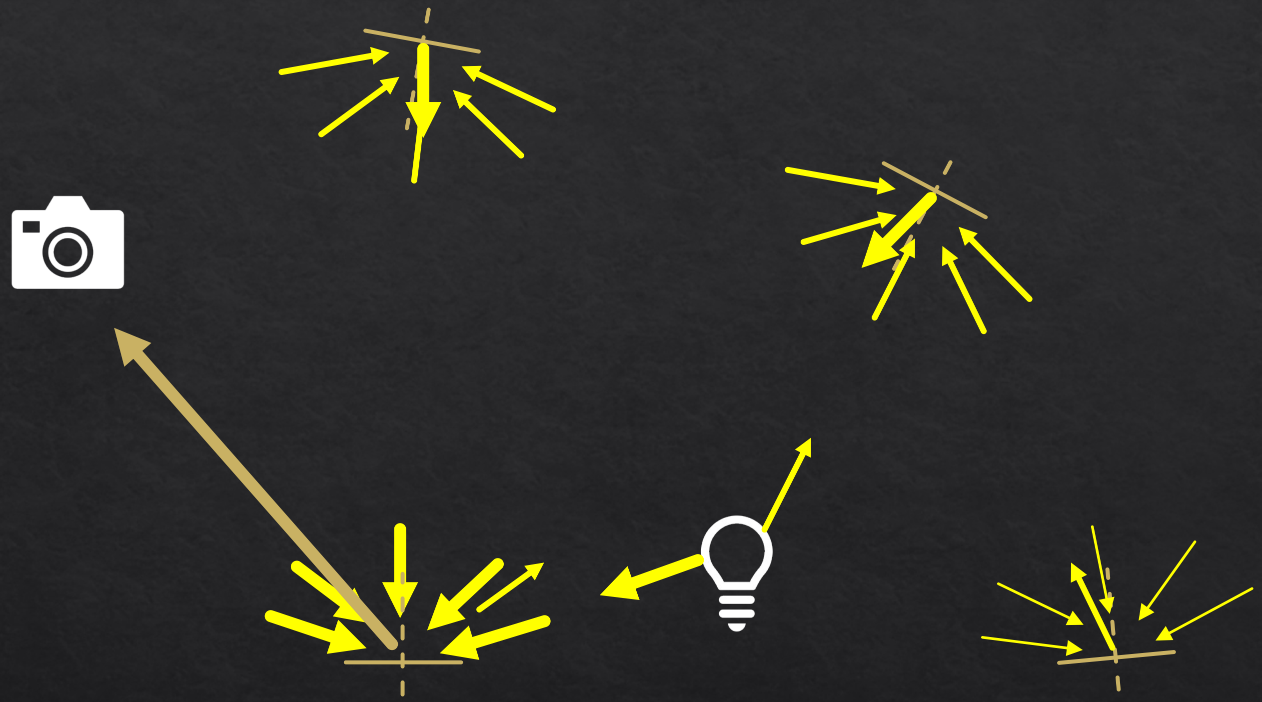
2) Trace each sample ray back to intersection

Estimate integrals recursively



1) Sample hemisphere at last bounce to camera
2) Trace each sample ray back to intersection
3) Sample those hemispheres
4) Recurse until k bounces
5) Use recursion results to estimate radiance

Estimate integrals recursively



k bounces with n samples each
= n^k samples per pixel

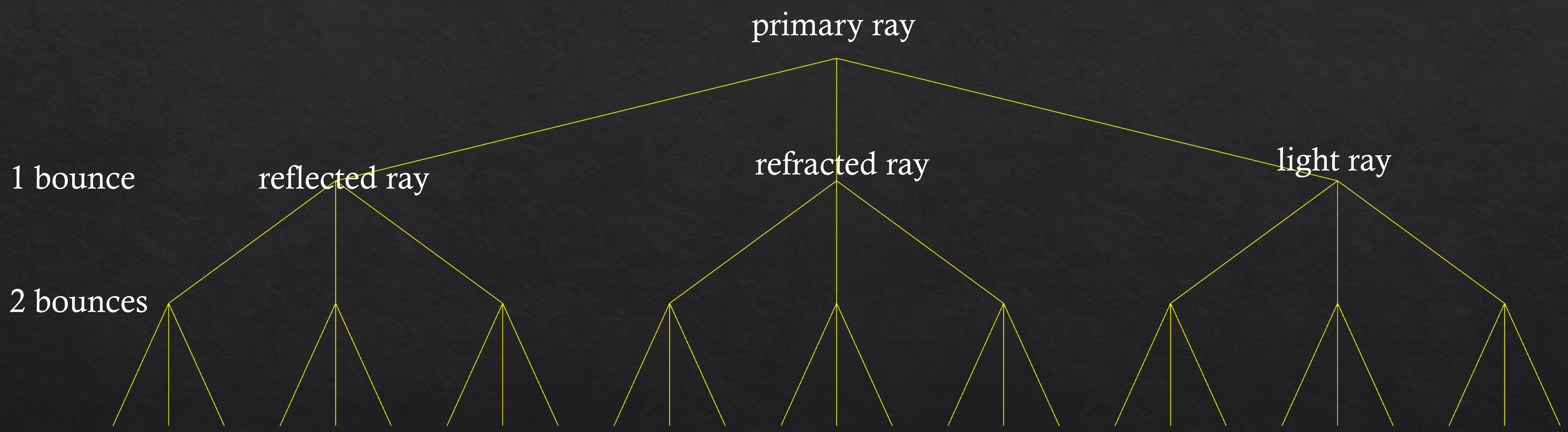
e.g. 8000 spp if n = 20 and k = 3

Let there be blur!

Numerical integration
- aperture
- time
- materials
- penumbra

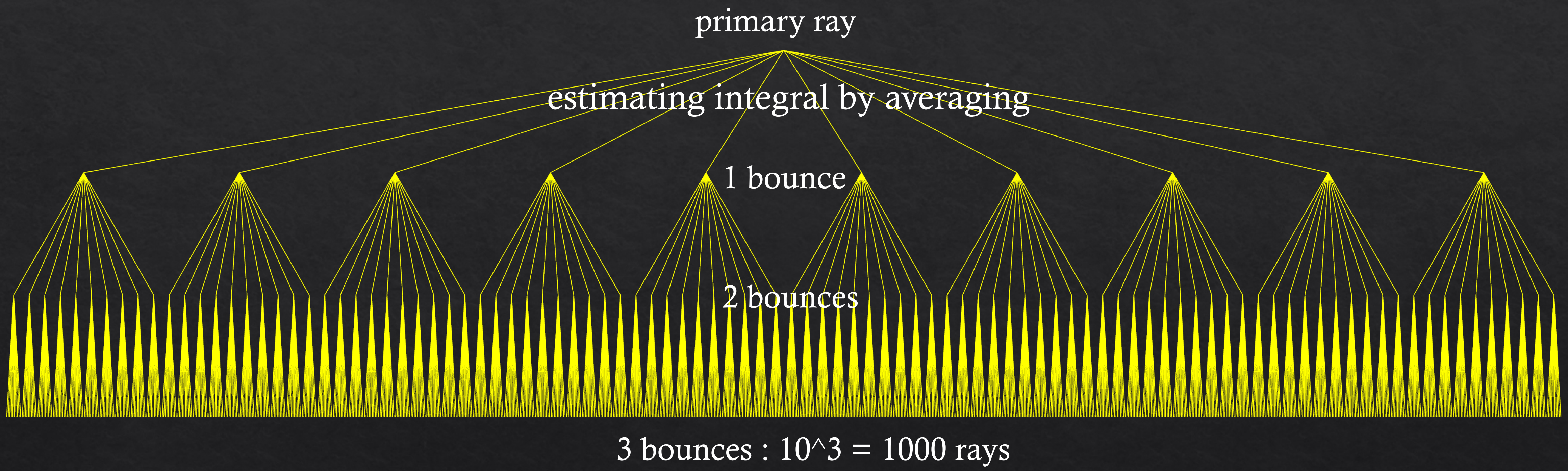
Distributed ray tracing [Cook et al 1984]

Whitted ray tracing – ray tree

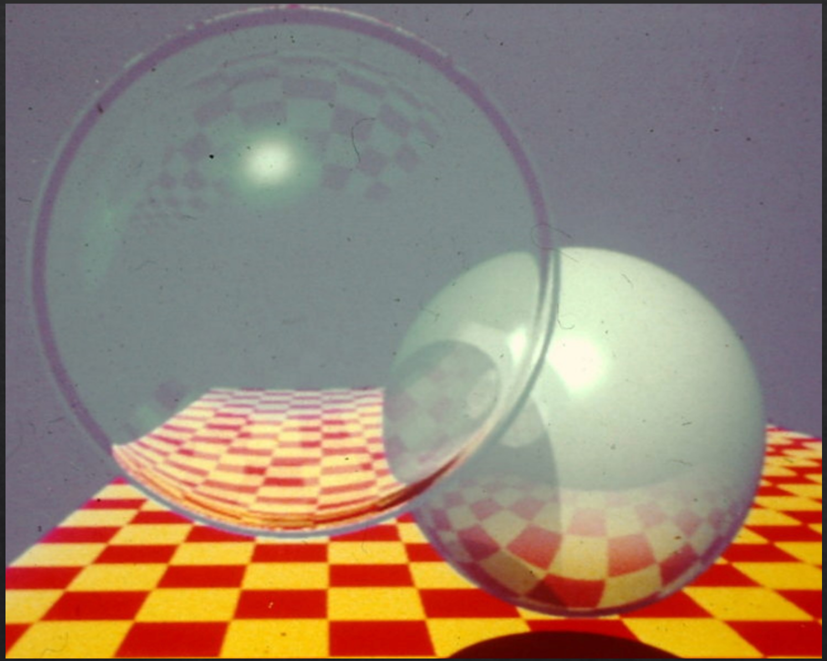
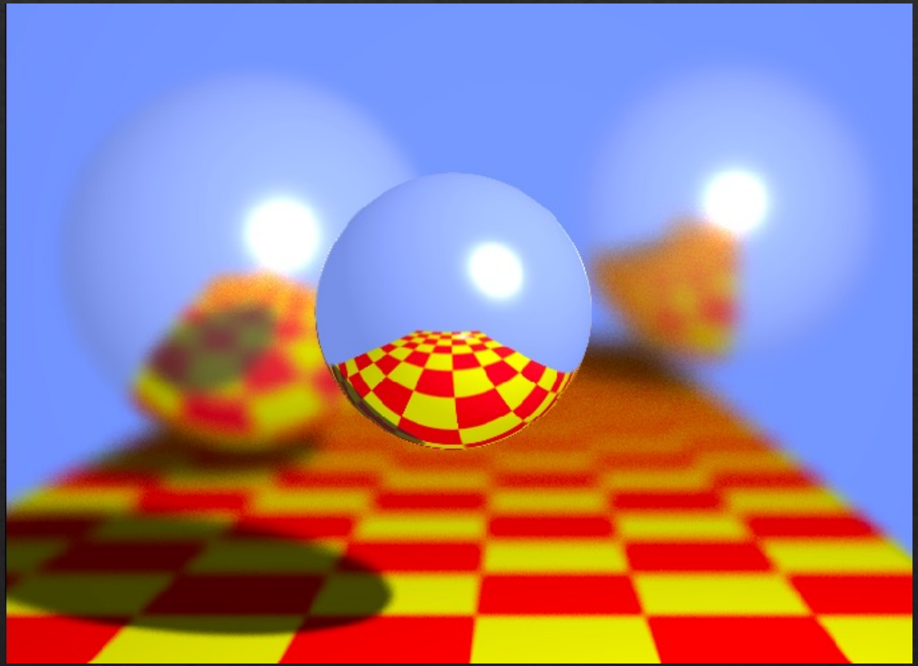
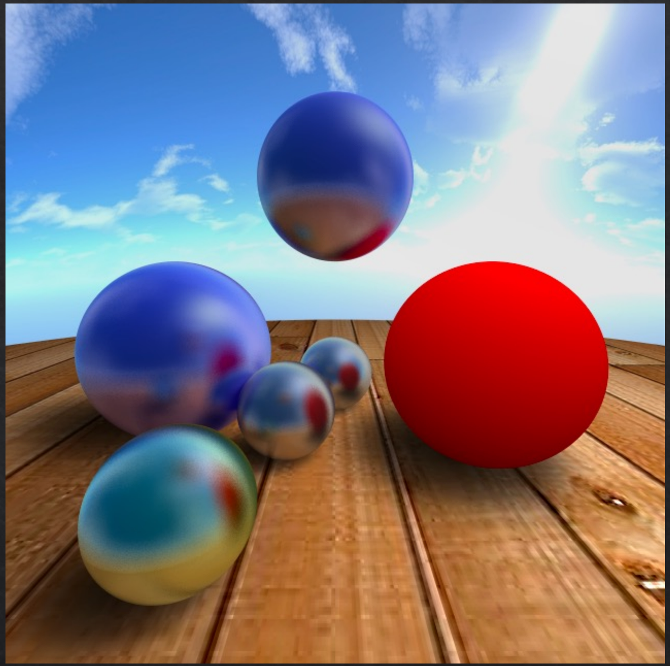


3 bounces : 3^3 = 27 rays

Distributed ray tracing – ray tree

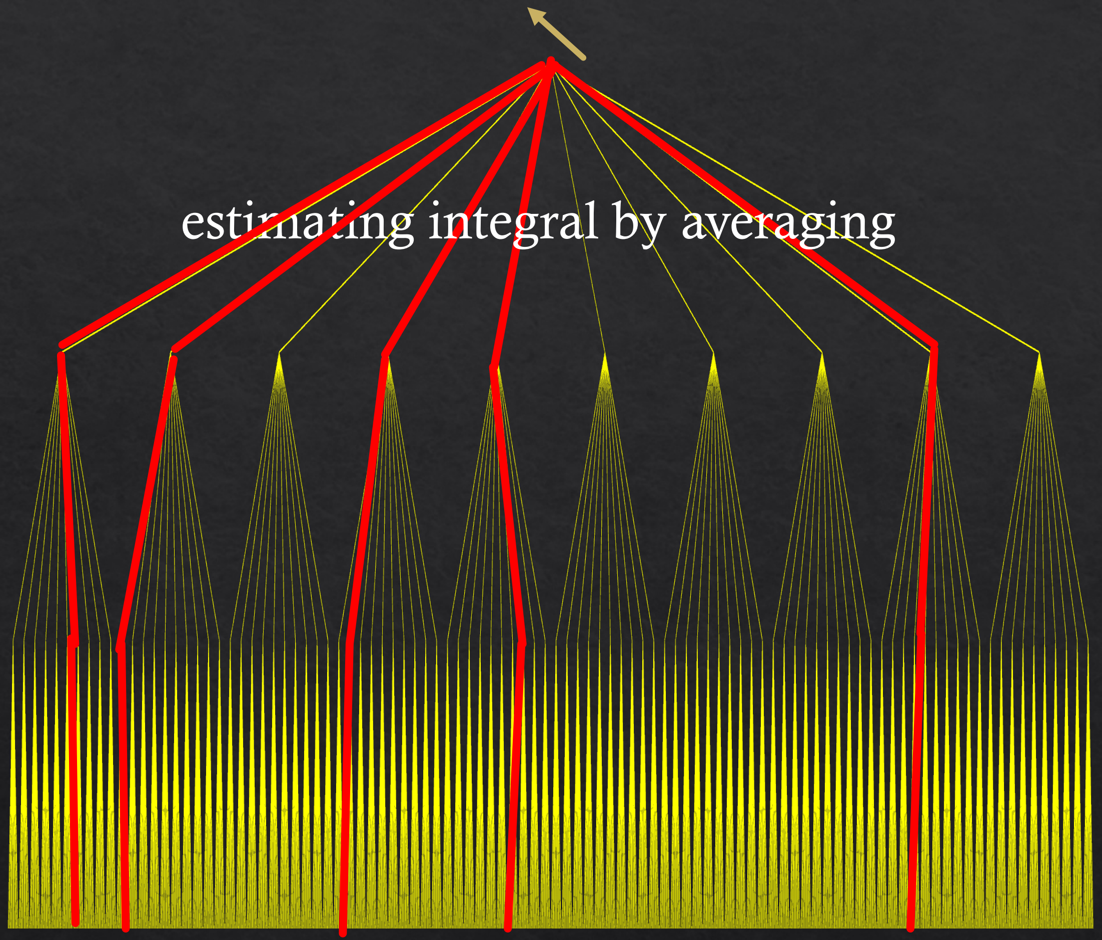


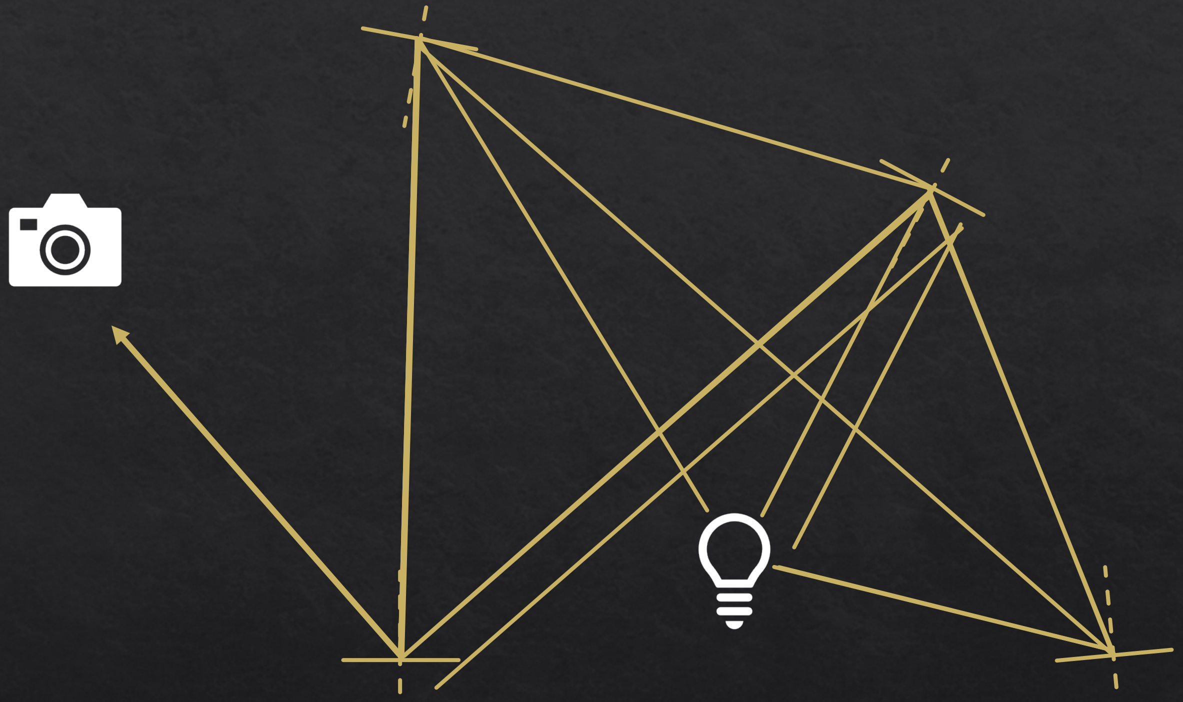
Helps, but expensive!

Better way to solve the rendering equation?

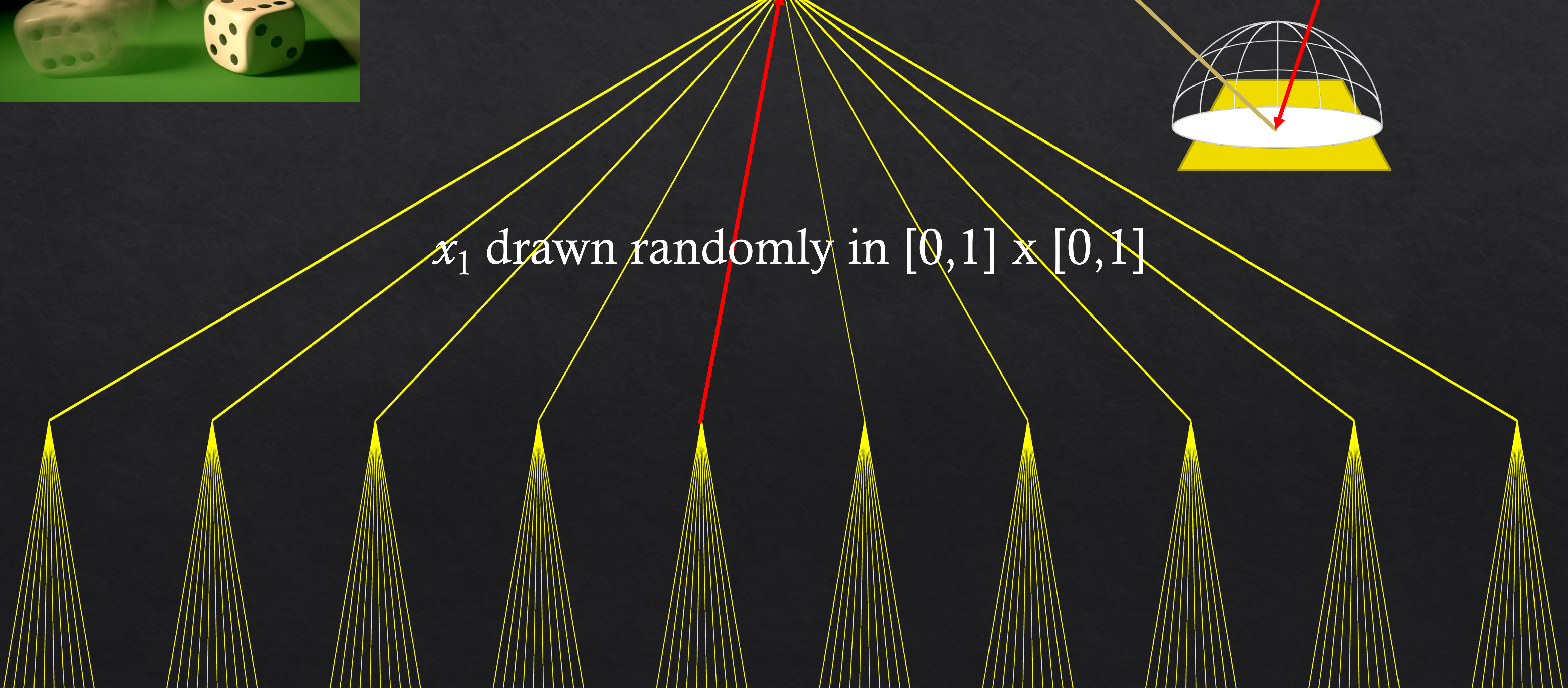
estimating integral by averaging





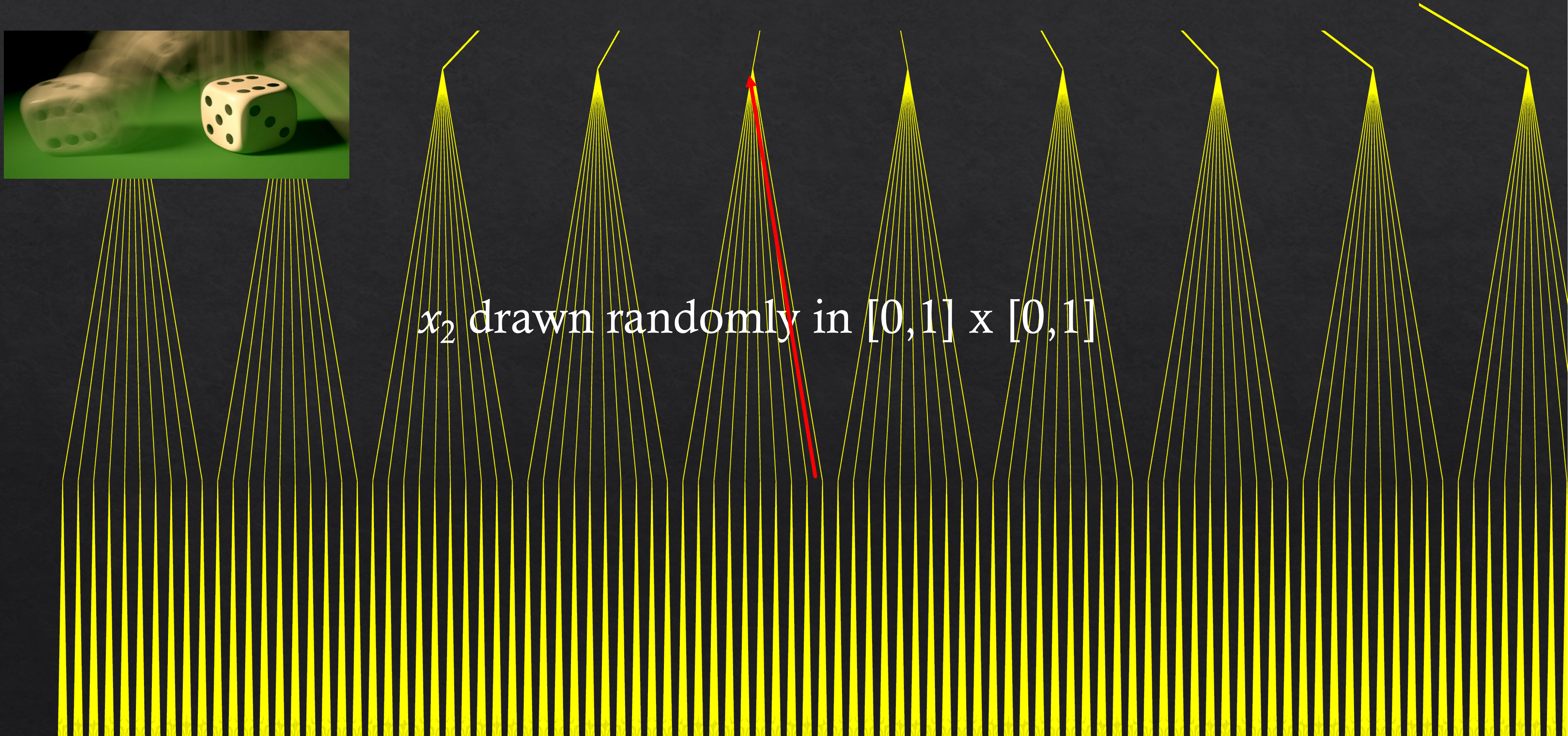
path tracing [Veach98]

Random sampling at each level

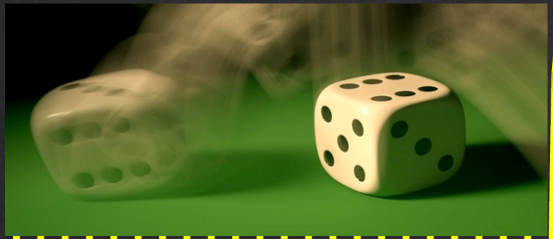


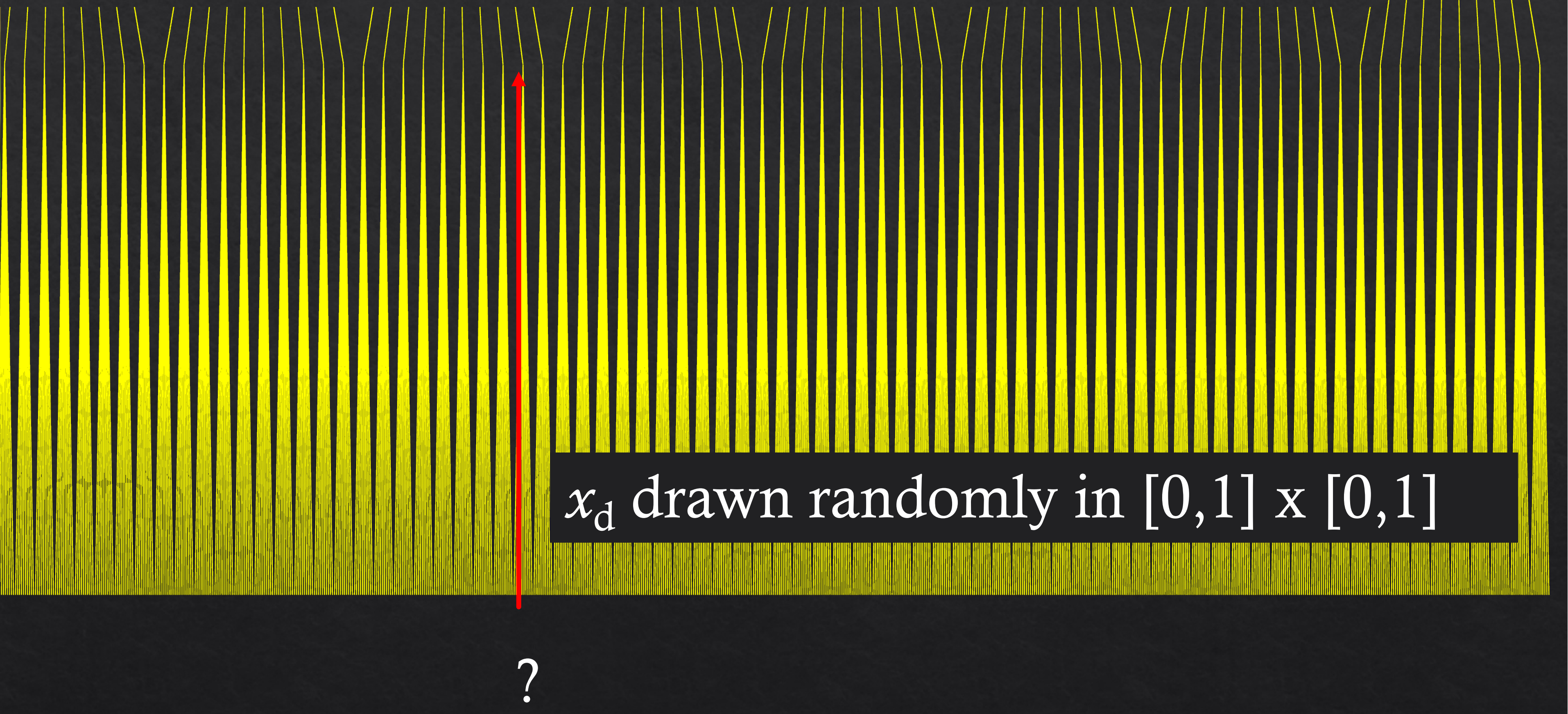
x_1 drawn randomly in [0,1] x [0,1]

Random sampling at each level



When to terminate?





When to terminate path?

Fixed depth *d*

When radiance is low

threshold

randomly

Russian roulette

https://twitter.com/DisneyAnimation/status/1146085535057715200

More bounces? depends on scene

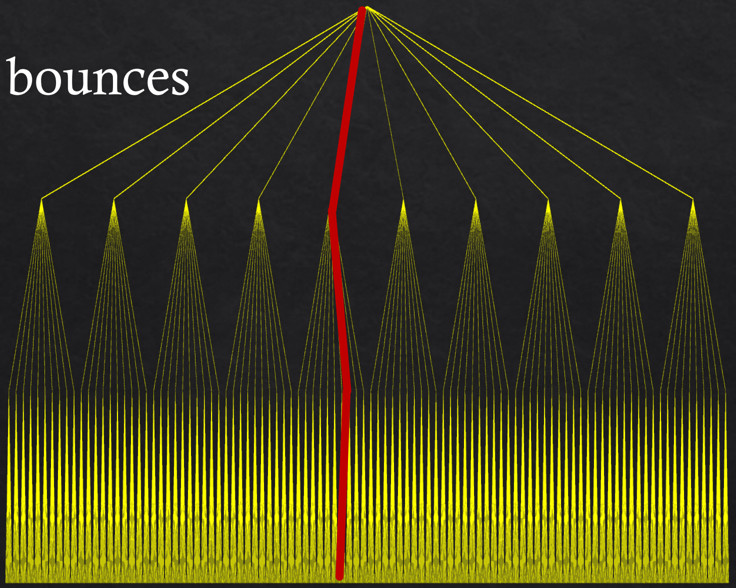
Path tracing each pixel - overview

pixel area — 2D

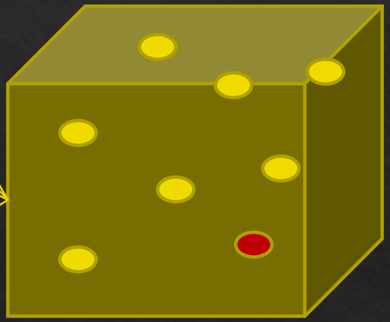
aperture — 2D

exp. time — 1D

bounces — d*2D

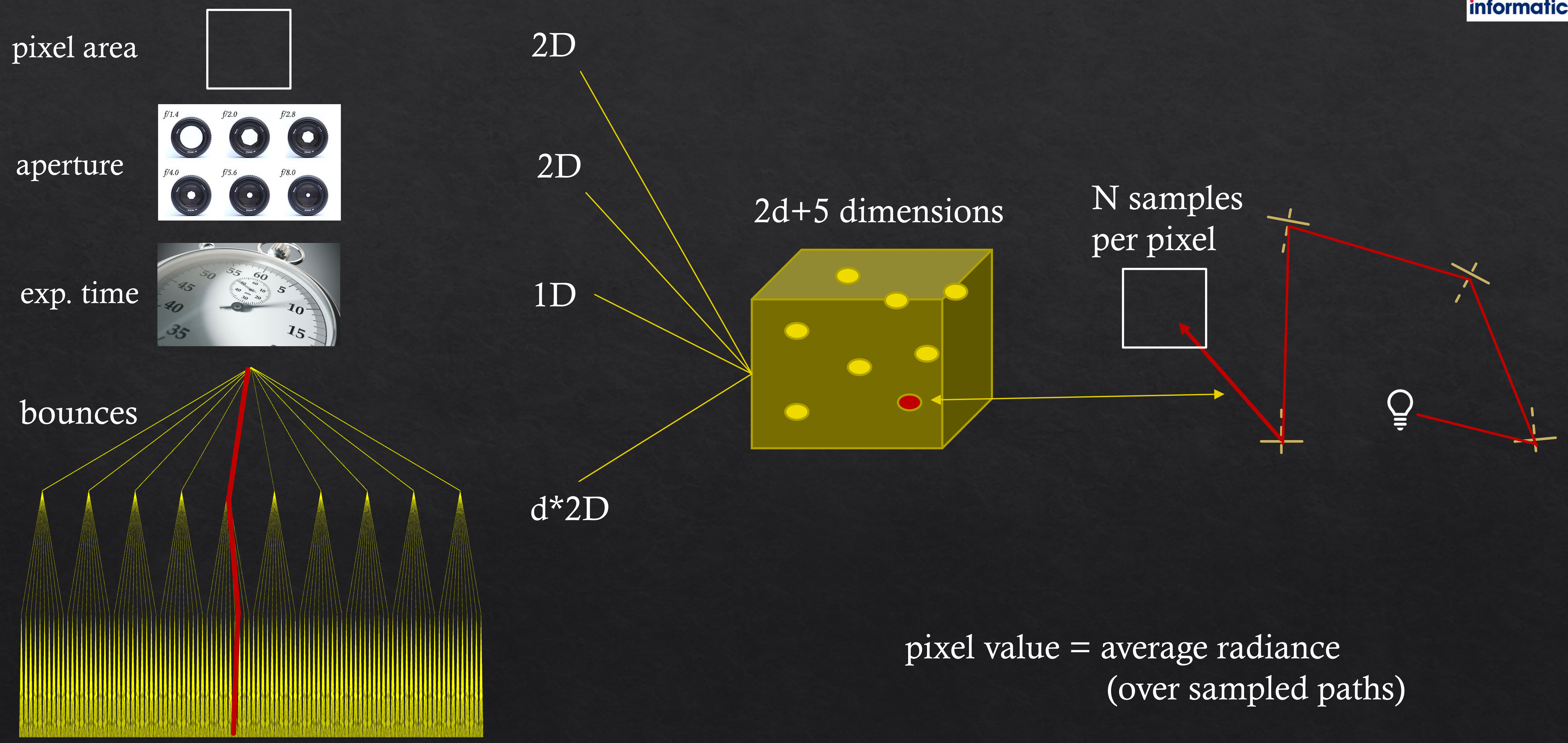


2d+5 dimensions



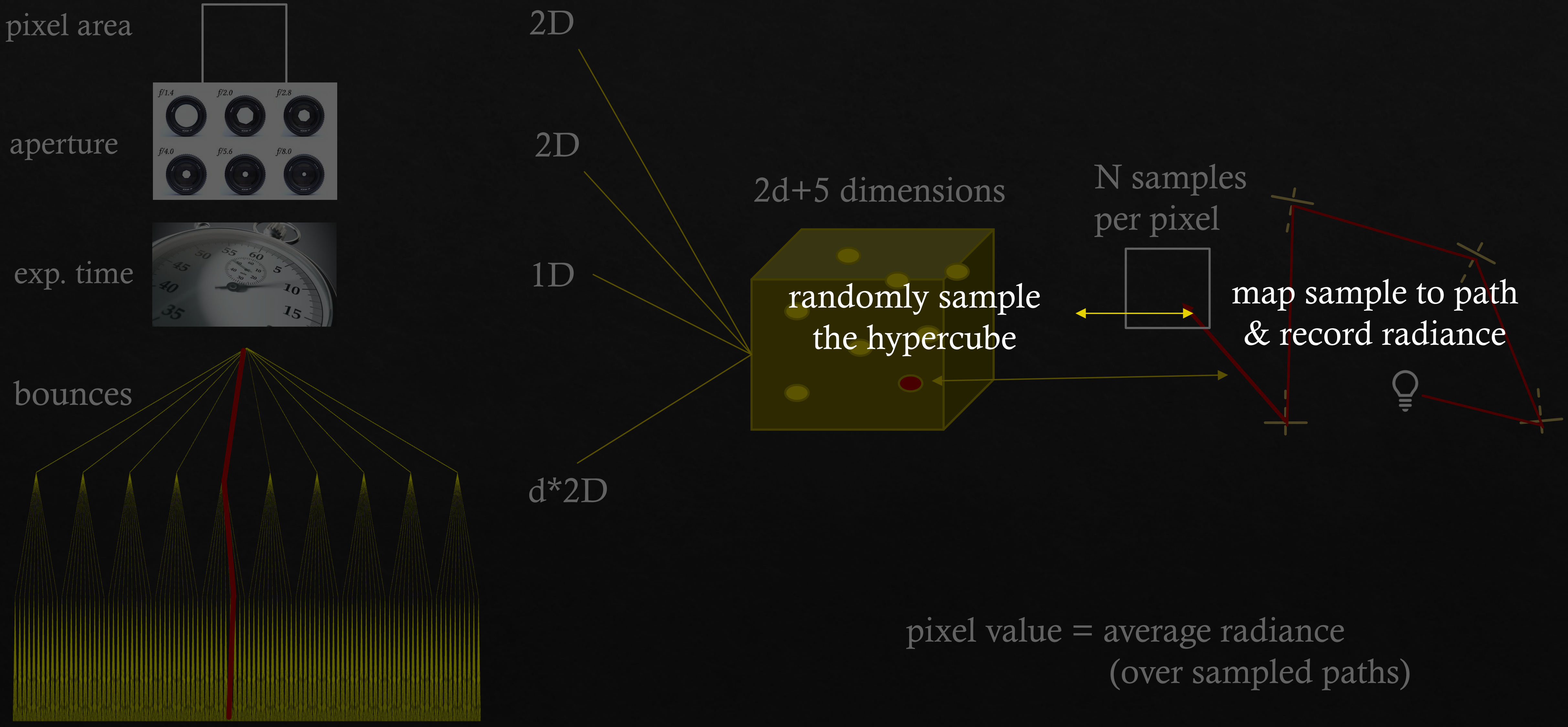
pixel value = average radiance
(over sampled paths)

Path tracing each pixel - overview



pixel value = average radiance
(over sampled paths)

Path tracing: mapping samples to paths



pixel value = average radiance
(over sampled paths)

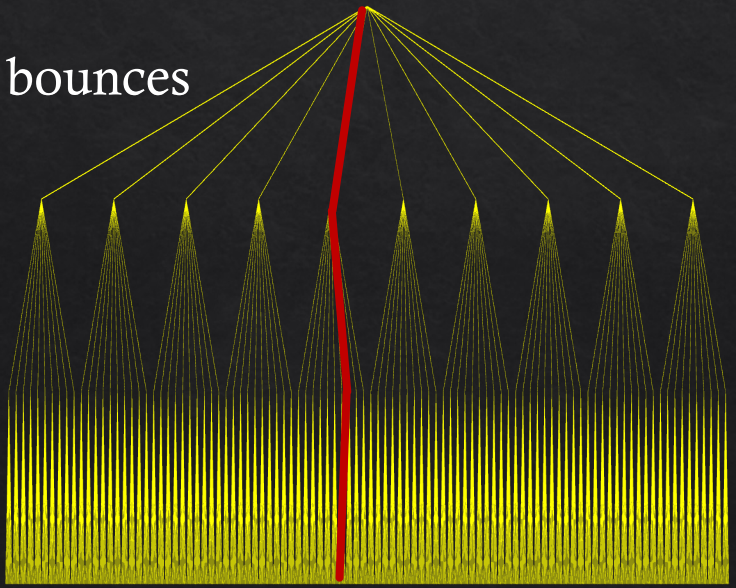
Path tracing: mapping samples to paths

pixel area — 2D

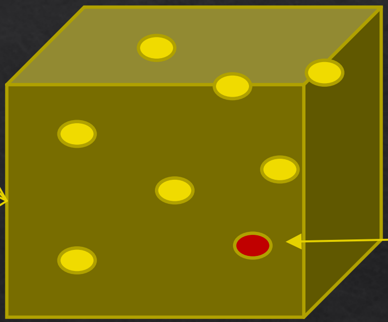
aperture — 2D

exp. time — 1D

bounces — d*2D



2d+5 dimensions



N samples per pixel

randomly sample the hypercube ⟷ map sample to path & record radiance

pixel value = average radiance (over sampled paths)

Path tracing - maths

$$I_p = \int_{\mathcal{P}} h_p(\mathbf{X}) \cdot f(\mathbf{X})\, \mathrm{d}\mathbf{X}$$

pixel average radiance

pixel filter weighting

$$f(\mathbf{X}) = L_e(\mathbf{x}_1)T(\mathbf{x}_1,\mathbf{x}_2)G(\mathbf{x}_1,\mathbf{x}_2) \cdot \left(\prod^{k-1} f_x(\mathbf{x}_i)T(\mathbf{x}_i,\mathbf{x}_{i+1})G(\mathbf{x}_i,\mathbf{x}_{i+1}) \right) \cdot W(\mathbf{x}_k).$$

[https://jo.dreggn.org/path-tracing-in-production/2019/index.html http://madebyevan.com/webgl-path-tracing/

This course so far …

content

assessment

This course so far …

This course so far …

content

assessment

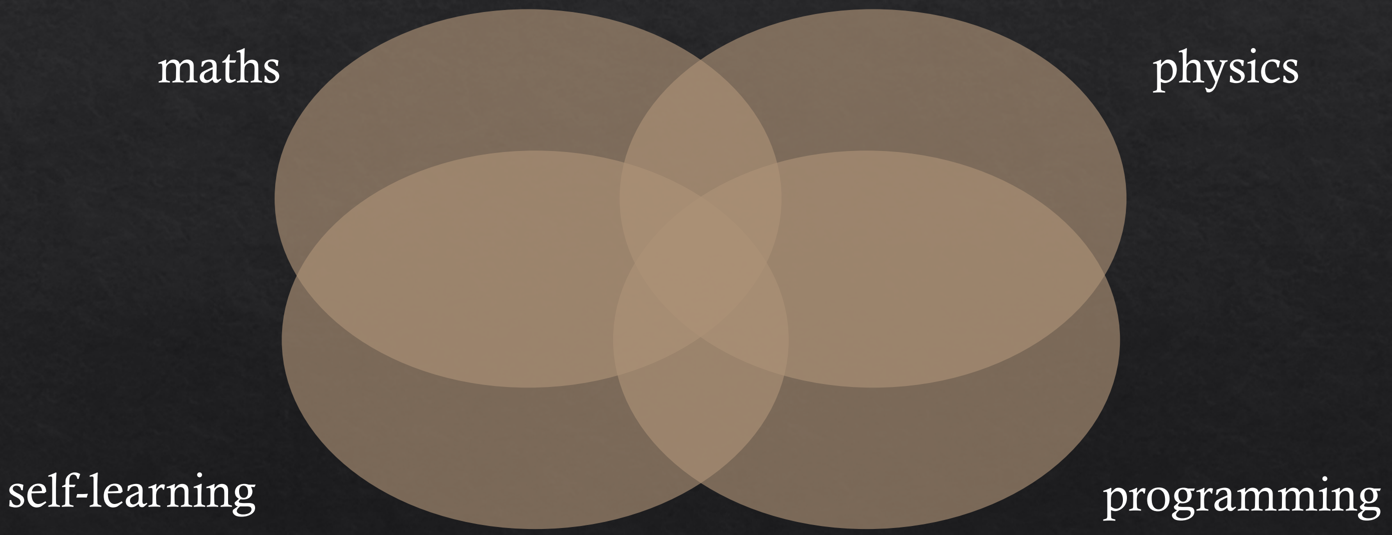
maths

physics

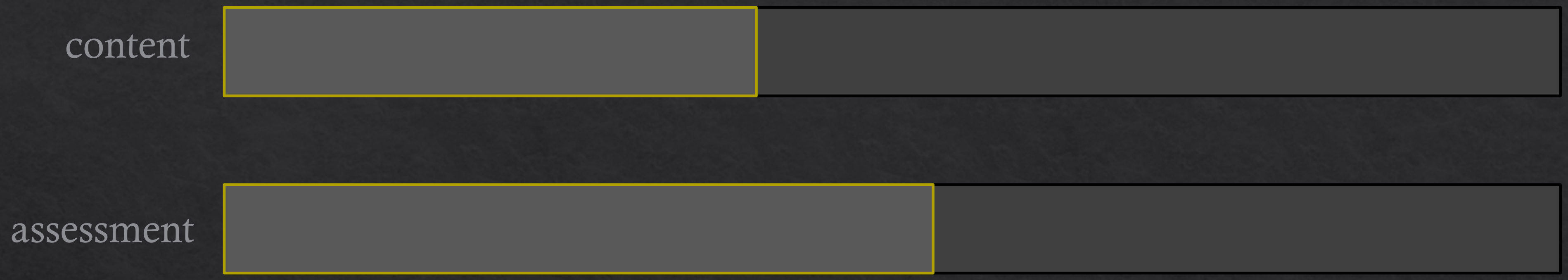
self-learning

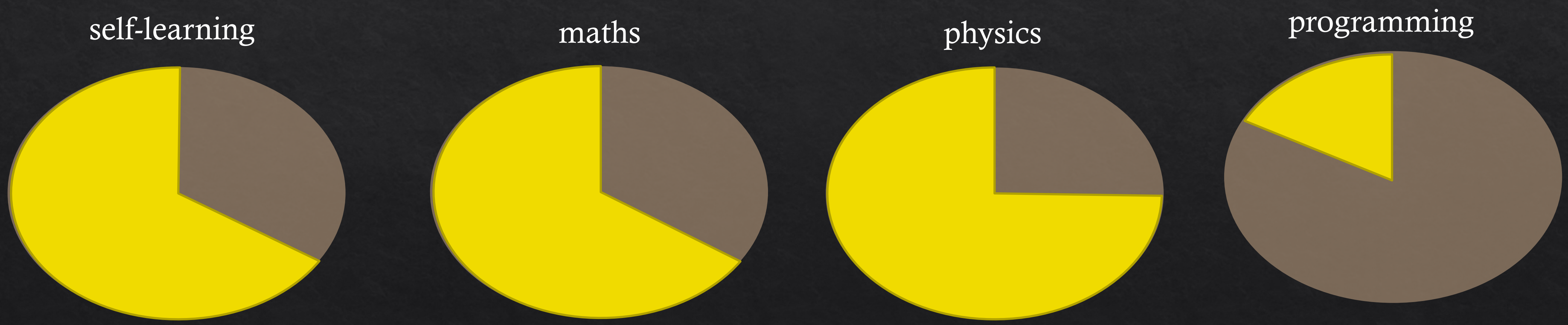
programming

This course so far …



This course so far …





This course

- v 1.0 (2007, Columbia University, NY)

- Evolved
    - current trends/needs (e.g. online resources, LLMs)
    - mixture of fun + skills (awareness vs career in CG)

- assessment and learning are not independent!

Feedback/appraisal

- Piazza (anonymous)
- email me (personal)
- student feedback
- nominate for teaching awards https://www.eusa.ed.ac.uk/whatson/awards/teachingawards

Quiz + feedback

1) Define radiance
2) Define irradiance
3) How would you obtain 1 from 2 and 2 from 1

Feedback on a scale of 1-10 (1-bad and 10-good)

1) Lectures are interesting
2) Lectures are difficult
3) I feel like I am learning, from this course
4) I am enjoying this course
5) Level of difficulty of tutorials
6) Recommendations for second half of the course (list one or two)
7) Describe (1-2 sentences) what changes you would recommend for material covered thus far, for the next offering of the course